

Comparative Analysis of ARIMA and GARCH Models for Forecasting Oil Prices and Exchange Rates in Nigeria

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Abstract

Forecasting future crude oil prices and exchange rates is essential because fluctuations in exchange rates and crude oil prices tend to have an impact on the economy. The study aimed at modelling and forecasting exchange rates and oil prices using some alternative models. The objectives of the study are to estimate suitable time series (ARIMA and GARCH) models for forecasting exchange rate in Nigeria; to estimate suitable time series (ARIMA and GARCH) models for forecasting crude oil prices in Nigeria and lastly, to compare the performance of the estimated models. The data used for the study were collected from the Central Bank of Nigeria Website and the United States Energy Administration. The data collected were analysed using the Augmented Dickey-Fuller (ADF) test, ARIMA, and GARCH. From the results of the analysis, the GARCH (1,2)-t emerged as the optimal model for modelling and forecasting monthly exchange rates in Nigeria while the ARIMA (2,1,0) emerged as the optimal model for modelling and forecasting monthly oil prices in Nigeria. Based on these findings it was recommended that policy makers may wish to adopt GARCH (1,2)-t and ARIMA (2,1,0) for forecasting exchange rate volatility and oil price respectively.

Keywords: Crude Oil, Exchange Rate, ARIMA, GARCH, ADF.

Introduction

In Nigeria, where government revenues, inflation, foreign investment, and economic stability are intimately tied to oil prices and exchange rates, accurate forecasting must be made to make effective policymaking. A key topic when discussing the global economy is foreign exchange (Etuk et al., 2016). According to Mandel et al. (2015), foreign exchange rate dynamics play a major role in modern macroeconomics. Making policy requires the use of exchange rate forecasting and modelling (Hina & Qayyum, 2015). The macroeconomic fundamentals of oil price, interest rate, wage, unemployment, and rate of economic growth are all significantly impacted by the exchange rate, making exchange rate forecasting critical (Ramzan et al., 2012). Trillions of dollars in different currencies are transacted daily on foreign exchange markets, which are among the most significant and expansive financial markets in the world (Khashei & Bijari, 2011). Trading occurs globally, twenty-four hours a day. The cost of one currency relative to another is known as the foreign exchange rate (CBN, 2016).

Crude oil is, without a doubt, one of the most significant commodities in the world today (Selvi et al., 2018). A large number of the items we use daily are manufactured and transported using oil. The oil-producing sector has a major impact on other industries. The expenses of producing other goods and even the state of the economy are greatly impacted by changes in the price of petroleum products (Fondo et al., 2021). Changes in the price of oil have a ripple effect on many aspects of our everyday life, including the availability of food, detergents, prescription medications, and home appliances, to mention a few. Like any other commodity, its price fluctuates in response to supply and demand in the market. As oil prices rise or fall, they will either have a detrimental influence on many economic sectors or a beneficial impact on others (Mensah, 2015). putting crude oil at the forefront of global economic policy discussions and one of the primary economic concerns.

Given the last two decades, it is clear that the financial crisis caused by the housing bubble harmed oil prices, which went from US\$ 133.88 per barrel in June 2008 to under \$40 per barrel within a few months of the crisis. Following that, the prices rose till they reached \$100 in 2014, but this did not persist for long, and they dropped below \$30 in 2016 as a result of the increased supply of crude oil. Due to the impacts of COVID-19, prices in 2020 decreased to their lowest point in 20 years, reaching \$ 16.55 in April. However, due to the situation in Ukraine, prices have increased once more. These shifts in the price of crude oil internationally undoubtedly have an impact on nations that rely significantly on imported crude oil (Nyongesa & Wagala, 2016; Rodhan & Jaaz, 2022; Selvi et al., 2018). Recently, COVID-19 hurt oil prices. The crude oil prices in 2020 decreased to their lowest point in 20 years, reaching \$16.55 in April. However, due to the Ukraine war, prices increased once more. These shifts in the price of crude oil internationally undoubtedly have an impact on nations that rely significantly on imported crude oil (Nyongesa & Wagala, 2016; Rodhan & Jaaz, 2022; Selvi et al., 2018).

Forecasting oil prices and exchange rates is relevant for Nigerian economic planning since the aforementioned variables have far-reaching implications for government revenue, inflation, investment, and general stability. In this study, an autoregressive integrated moving average (ARIMA) and a Generalized Autoregressive Conditional heteroskedasticity (GARCH) model, which statistically predicts future points with respect to time-varying phenomena, are considered. There are three components to an ARIMA model: a combination of autoregressive (AR), integrated (I), and moving-average (MA) models. Autoregression revolves around regressing the variable of interest (in this case, the price of oil or exchange rate) on its prior terms. The "I" part of the model is generally applied when the data in the sample are non-stationary (i.e., the joint probability distribution of the variables under consideration changes over time). The moving-average model states that the output variable linearly depends on a stochastic term's present and past values.

Gaspar and Mbwapbo (2023) address the problem of predicting crude oil prices in a situation of unusual circumstances. Box-Jenkins methodology was used to analyze the monthly dynamics of the Brent oil price from January 2002 to February 2022. Data were

first differenced to achieve stationarity, and then ACF and residual diagnostics were utilized to choose models that were used for analysis. The performances of various models were evaluated, and ARIMA (0, 1, 1) was found to be the best model for forecasting crude oil prices. This study further reveals that despite the coronavirus and the Ukraine war having a considerable impact on crude oil prices, such a model is still capable of capturing the underlying volatility in crude oil prices.

Xiang (2022) carried out a study aimed at comparing the autoregressive integrated moving average model and the combination model of autoregressive integrated moving average model-generalized autoregressive conditional heteroskedasticity (ARIMA-GARCH) for analyzing and forecasting international oil prices, study shows that the combination model of ARIMA (1,1,0)-GARCH (1,1) is more suitable for short-term forecasting of International oil prices with higher accuracy than the MAPE of forecasting has reduced from 1.549% to 0.045% and the RMSE of forecasting has reduced from 1.032 to 0.071.

Yahaya et al. (2021) compare the performance of Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroskedasticity models in forecasting Crude Oil Price data as obtained from the CBN 2019 Statistical Bulletin. They forecasted that Crude Oil Prices play an important role in decision-making for the Nigerian government and all other sectors of the economy. Crude Oil Prices are volatile time series data, as they have huge price swings in a shortage or an oversupply period. In this study, they used two-time series models, which are Box-Jenkins Autoregressive Integrated Moving Average and Generalized Autoregressive Conditional Heteroscedasticity models in modelling and forecasting Crude Oil Prices. The statistical analysis was performed by the use of a time plot to display the trend of the data, Autocorrelation Function, Partial Autocorrelation Functions, and Dickey-Fuller test for stationarity, Forecasting was done based on the best-fit models for both ARIMA and GARCH models. The result shows that ARIMA (3, 1, 2) is the best ARIMA model to forecast monthly Crude Oil Prices and also found GARCH (1, 1) model is the best GARCH model and uses a specified set of parameters, GARCH (1, 1) model is the best fit for our concerned data set.

AL Shammre and Chidmi (2023) investigate the forecasting accuracy of alternative time series models when augmented with partial least-squares (PLS) components extracted from economic data, such as Federal Reserve Economic Data, as well as Monthly Database (FRED-MD). Our results indicate that PLS components extracted from FRED-MD data reduce the forecasting error of linear models, such as ARIMA and SARIMA, but produce poor forecasts during high-volatility periods. In contrast, conditional variance models, such as ARCH and GARCH, produce a more accurate forecast regardless of whether or not the PLS components extracted from FRED-MD data are used.

Suleiman et al. (2023) identified the best ARIMA time series model for monthly crude oil prices in Nigeria from 2006 to 2020. At first, the stationary condition of the data series is observed by time plot, Autocorrelation Function and Partial Autocorrelation Function Plots, and then confirmed using Kwiatkowski-Phillips-Schmidt-Shin and Phillips-Perron test statistic, which has been found that the crude oil price series is non-stationary. After taking

the first difference of logarithmic values of the data series, the crude oil prices data become stationary. The Box-Jenkins four-step iterative methodology comprising model identification, model fitting, diagnostic and forecasting is also applied to the crude oil prices data. Two optimal time series models were selected, namely ARIMA (2,1,1) and ARIMA (3,1,1), based on AIC, BIC and HQC. Thus, based on the criteria of mean square error, root mean square error, and mean absolute error, the ARIMA (3,1,1) model best fits the data with minimum values of predictive measures.

Similarly, Odukoya and Adio (2022) carried out a study with the title: Time Series Analysis of Exchange Rate Nigerian Naira to US Dollar. Their study revealed that ARIMA (1, 1, 0) is an appropriate model for forecasting Nigerian Naira to US dollar exchange rate. Oyengu *et al.* (2019) model the exchange rate of Nigeria against four major currencies using the ARIMA model. The result of the analysis revealed that the four major currencies (Dollar, Pounds, Euro and Swiss Franz) times series data were not stationary at their original form and followed ARIMA (1,2,1); ARIMA (2,2,1); ARIMA (2,2,1) and ARIMA (2,2,2) respectively. Bala *et al.* (2013) use variants of GARCH volatility models to examine monthly exchange-rate returns for three major currencies in the Nigerian FX markets: The Naira/USD, Euro and BPS. Their findings reveal that TGARCH is the best-fitting model for the Euro, while ARCH and PARCH (1,1) are the best-fitting models for BPS return and Naira/USD returns, and IGARCH was suitable for the USD return model with volatility breaks.

Schmidt (2021) investigated the volatility forecasting performance of symmetric and asymmetric GARCH models on Nordic indices during COVID-19. The models examined in this paper are GARCH, EGARCH, GJR and APARCH, and the forecasting performance is evaluated by several statistical performance measures. The results of this paper are that the symmetric GARCH (1,1) on average has the worst forecasting performance during a crisis. However, the difference between the predictability of the models is, in practice, small. The superior forecasting models are the GJR (1,1) and EGARCH (1,1) when forecasting a crisis on Nordic indices according to the evaluation measures. This is because the asymmetric extensions of these models enable them to capture the more present leverage effect in the event of a crisis.

The Autoregressive Conditional Heteroskedasticity (ARCH) model is generalized in time series by the GARCH model, whereas the ARIMA model is a generalized version of the ARMA model. The models which relate to the order of the autoregression, the integration as in ARIMA and the moving average are commonly referred to as ARIMA (p, d, q) and GARCH (p, q) models, where p, d, and q are integers greater than or equal to zero. These two models are methods of stochastic modelling that capture the time-dependent structure embedded in the time series data on crude oil price and exchange rate and can be used to study the dynamics and evolution of these variables. Conditional heteroscedasticity (the shifting variance character of the data) is accommodated by the ARIMA model, also known as the Box-Jenkins Methodology, and the ARCH models, with their extension to GARCH models as developed by Engle (1982) and Borellslev (1986), respectively. Both the

exchange rate and the price of the energy commodity are very erratic over time. Forecasting future crude oil prices is essential because fluctuations in exchange rates and crude oil prices have an impact on the stock market and other industries. "Which method between the Box-Jenkins and GARCH performs better in forecasting crude oil prices and exchange rates, which are of high volatility?" is the question that this study will attempt to answer.

Materials and Methods

The rationale for selecting ARIMA and GARCH over other Time Series Models for forecasting oil prices and exchange rates is appropriate and necessary due to the inherent volatility and structural characteristics associated with the data. They can effectively capture salient characteristics of economic and financial data. The GARCH and ARIMA models are used in particular because of the following characteristics of financial time series data:

- i) **Fat Tails:** A fat-tailed distribution is a probability distribution that exhibits large skewness or kurtosis. When the distribution of the financial time series is compared with the normal distribution, fatter tails are observed.
- ii) **Volatility Clustering:** The second stylized fact is periods of volatility clustering, which means that large and small values in the log returns tend to occur in clusters, i.e. large changes tend to be followed by large changes, and small changes tend to be followed by small changes. When volatility is high, it is likely to remain high; when it is low, it is likely to remain low. Volatility clustering is nothing but the accumulation or clustering of information. This feature reflects the fact that news is clustered over time (Engle, 2004).
- iii) **Mean reverting:** The variance of the financial time series data often deviates from the mean. In most cases, the reversion is not by a constant factor, a property known as heteroscedasticity
- iv) **Ability to Model Non-Stationary Data:** ARIMA includes a differencing step (the "Integrated" component) that helps transform non-stationary data into stationary data, making it suitable for economic and financial data that often exhibit trends.
- v) **Captures Autocorrelation:** The Autoregressive (AR) and Moving Average (MA) components effectively model relationships between past and present values, improving forecasting accuracy.

Source of Data Collection

The study used secondary sources of data, which were obtained from the Central Bank of Nigeria Website and the United States Energy Administration. The documentary method of data collection was employed in this study from January 2000 to April 2024.

After the data was collected, the next step involved cleaning, organizing, and preparing it for analysis. The process started with data cleaning, where issues like missing values, duplicates, errors, and inconsistencies were addressed. Linear interpolation technique was used for the missing data.

After cleaning, the next step was data transformation and preprocessing. This step included scaling numerical values through normalization and standardization to ensure all features were on a comparable scale. Log transformations to reduce skewness and used first-order differencing to make the data stationary, which is essential for the models was addressed. Before diving into the analysis, exploratory data analysis (EDA) was conducted to understand the dataset better. This involved summarizing key statistics like mean, median, standard deviation, and correlation coefficients to get a sense of the data's distribution and relationships. Visual tools like histograms and time series graphs to identify patterns, trends, and potential outliers were used. Outliers were detected using the Interquartile Range (IQR) method.

With the data fully prepared, the EViews statistical technique was used to analyse the data. Finally, I evaluated and interpreted the models to ensure their accuracy and reliability. I assessed their performance using metrics like Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Absolute Percentage Error (MAPE).

Techniques for Data Analysis

The study employed generalized autoregressive conditional heteroscedasticity (GARCH) and autoregressive integrating moving average (ARIMA) models as a technique for data analyses.

Returns on Stock Market Capitalization

The monthly exchange rate and daily crude oil prices were converted to returns series by this simple operation

$$r_t = \ln \left(\frac{p_t}{p_{t-1}} \right) \text{-----} 3.1$$

Unit Root Test

The Augmented Dickey-Fuller (ADF) was employed in testing the stationary of the returns on stock market capitalization. The ADF constructs a parametric correction for higher-order correlation by assuming that the time series data follow an autoregressive of order p process and adding p lagged difference terms of the dependent variables y to the right-hand side of the test regression as follows:

$$\Delta y_t = \alpha y_{t-1} + x_t' \delta + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + \dots + B_p \Delta y_{t-p} + v_t \text{-----} 3.2$$

Where x_t are optional exogenous regressions, which may consist of a constant or constant trend.

The ADF t-test null hypothesis is given by:

$H_0: \theta = 0$ implying that the data needs to be stationary by taking its difference. Against the alternative hypothesis:

$H_1: \theta < 0$, implying that the data is trend stationary and needs to be analyzed using time trends in the regression model instead of differencing the data.

The test statistic is a conventional t-ratio for a:

$$t_a = \frac{\hat{a}}{se(\hat{a})} \text{-----} 3.3$$

Where $\hat{a}$ is the estimate of a, and $se(\hat{a})$ is the coefficient standard error.

ARCH (P) Model

Engle (1982) was the first to introduce the conditional heteroscedasticity concept and changed the classical assumption of constant variance in time series models. He suggested that the autoregressive conditional heteroscedasticity (ARCH) model can allow the volatility to change over time as a function of past squared errors, leaving the unconditional variance stable. Simply, the idea of the ARCH model is that the shock or the errors ε_t are serially uncorrelated but dependent, and the dependence of ε_t can be described by a function of its squared and lagged values.

The mathematical formula for this model is as follows:

$$r_t = \mu_t + \varepsilon_t, \varepsilon_t = \sigma_t z_t, z_t \sim N(0,1) \text{-----} 3.4$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 \text{-----} 3.5$$

where $\omega > 0$, $\alpha_i \geq 0$, $i = 1, \dots, p$, and $p > 0$ is the order of the ARCH model, ω represents the average values of σ_t^2 , z_t is white noise with mean zero and variance 1. The μ_t is the appropriate structure explaining the mean equation. The ARCH coefficients α_i must satisfy stationary conditions to ensure that the unconditional variation exists. If $\sum_{i=1}^p \alpha_i < 1$, the ARCH model is weakly stationary with constant unconditional variables:

$$\sigma^2 = \frac{\omega}{1 - \sum_{i=1}^p \alpha_i} \text{-----} 3.6$$

The ARCH model of order 1 can be written as follows

$$\varepsilon_t = \sigma_t z_t \text{-----} 3.7$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 \text{-----} 3.8$$

The ARCH-LM Test

Engle (1982) proposed the ARCH LM test. The test begins with the estimation of the autoregressive model as specified in the equation below;

$$y_t = \alpha + \delta_i y_{t-i} + \varepsilon_t \text{-----} 3.9$$

where $\varepsilon_t \sim (0, \sigma^2)$

y : is the rate of return of the series,

α : is the constant term

δ : is the coefficient of the return series; and

ε : is the error term

The estimated residual is obtained from the equation in 3.9, then the squared of the estimated residual is regressed on its lag as follows:

$$\varepsilon_t^2 = y_0 + y_1 \varepsilon_{t-1}^2 + \dots + y_q \varepsilon_{t-q}^2 \text{-----} 3.10$$

The ARCH(q) model is

$$\sigma_t = y_0 + y_1 \varepsilon_{t-1}^2 + \dots + y_q \varepsilon_{t-q}^2 + y_q \varepsilon_{t-q} + \sum_{i=1}^q y_i \varepsilon_{t-i} + \varepsilon_t \text{-----} 3.11$$

$H_0 = 0$, while $H_i \neq 0$

The test statistics for the null hypothesis are the F-test and R^2 tests.

If the probability values (p-values) of these tests are smaller than any of the traditional levels of statistical significance (10%, 5%, and 1%), the null hypothesis that there are no ARCH effects is rejected.

Standard GARCH (p, q) Model

The ARCH approach was developed by Engle and has been applied extensively to time series data in the financial and economic domains. Nonetheless, numerous drawbacks of the model were discovered. The ARCH model, for instance, assumes that the conditional variance is equally affected by positive and negative shocks. Also, it is difficult to manage the existence of negative variance because it depends on the square of the previous shocks, the lag duration, and a large number of parameters. Bollerslev (1986), therefore, proposed the generalized ARCH, or so-called GARCH model, to overcome this problem by letting the current volatility depend on both the p previous squared innovations and the first q past volatility. The standard GARCH model is written as:

$$r_t = \mu_t + \varepsilon_t, \varepsilon_t = \sigma_t z_t, z_t \sim N(0,1) \text{-----} 3.12$$

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 \text{-----} 3.13$$

Where 3.10 and 3.11 are the mean and variance equation respectively, where $\omega > 0$, $\alpha_i \geq 0$, $i = 1, \dots, p$ and $\beta_j \geq 0$, $j = 1, \dots, q$, are sufficient conditions to ensure that the conditional variance $\sigma_t^2 > 0$. Also, μ_t is the appropriate structure explaining the mean equation, ω represents the average values of σ_t^2 , z_t is white noise with mean zero and variance 1. r_t is the continuous compounding log return series. The parameters α_i represents the ARCH effect, and β_j represents the GARCH effect. In addition, to achieve the stationarity requirement in GARCH models, the summation of $\sum_{i=1}^p \alpha_i + \sum_{j=1}^q \beta_j$ must be less than one. This summation reflects the persistence of innovations (shocks) to the volatility, meaning that the impact of a volatility shock disappears over time at an exponential rate.

Clearly when $q = 0$ the GARCH model will become the ARCH model. Hence, as its name suggests, GARCH is the generalization of the ARCH model. Therefore, the GARCH model of order one can be written as follows:

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \text{-----} 3.14$$

However, the GARCH (1,1) model is weakly stationary if $\alpha_1 + \beta_1 < 1$, in this case, the unconditional variance can be as follows:

$$var(\varepsilon_t) = \frac{\omega}{1 - \alpha_1 - \beta_1}.$$

On the other hand, the unconditional variance is infinite if $\alpha_1 + \beta_1 = 1$. Therefore, the GARCH model with $\alpha_1 + \beta_1 = 1$ is named the integrated GARCH (IGARCH) model.

Autoregressive Integrating Moving Average Model

Autoregressive (AR) Model

In the Autoregressive model of order p denoted by $AR(p)$, the future values of a variable are assumed to be a linear combination of p -past observations and a random error together with a constant term. Mathematically, the $AR(p)$ model can be expressed as:

$$y_t = c + \sum_{i=1}^p \phi_i y_{t-i} + \varepsilon_t \text{-----} 3.15$$

Given that

$$\sum_{i=1}^p \phi_i y_{t-i} = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t \text{-----} 3.16$$

Where,

y_t and ε_t are respectively the actual value and random error (random shock) of a white noise process at time t ; $\phi_i (i = 1, 2, \dots, p)$ are the model parameters, and c is the constant. The integer constant p is known as the order of the model. At times, the constant is omitted.

Moving Average (MA) Model

The moving average of order q given as the $MA(q)$ model uses past errors as the explanatory variable.

Mathematically, the $MA(q)$ is given by:

$$x_t = \mu + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t \text{-----} 3.17$$

Where

$$\sum_{i=1}^q \theta_i \varepsilon_{t-i} = \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q} \text{-----} 3.18$$

Where

μ is the mean of the series; $\theta_i (i = 1, 2, \dots, q)$ are the model parameters and q is the order of the model, the random shocks (ε_t) are assumed to be a white noise process that is; a sequence of independent and identically distributed random variables with mean zero and constant variance σ^2 .

Autoregressive Moving Average (ARMA) Model

The ARMA model consists of two parts: an AR part and an MA part. The model is usually then referred to as the ARMA (p, q) model, where p is the order of the autoregressive part and q is the order of the moving average part. The mixed model is given as:

$$y_t = \delta + \phi_1 y_{t-1} + \dots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \dots - \theta_q \varepsilon_{t-q} \text{-----} 3.19$$

or using the B-operator

$$\phi(B)y_t = \delta + \phi(B)\varepsilon_t \text{-----} 3.20$$

The model is referred to as the ARMA (p, q) scheme. The ARMA model is both stationary and invertible.

Autoregressive Integrated Moving Average (ARIMA) Model

In the autoregressive integrated moving average (ARIMA) model, a non-stationary time series is made stationary by applying finite differencing of the data points. The mathematical formulation of the ARIMA (p, d, q) model using lag polynomials is given as:

$$\phi(L)(1-L)^d y_t = \phi(L)\varepsilon_t \text{-----} 3.21$$

This can be expanded as:

$$\left(1 - \sum_{j=1}^p \phi_j L^j\right)(1 - L)^d y_t = \left(1 + \sum_{j=1}^q \theta_j L^j\right) \varepsilon_t \text{-----} 3.22$$

Here, p, d, and q are the integers greater than or equal to zero. And refer to the order of the autoregressive integrated moving average parts of the model respectively. The integer d controls the level of differencing.

Box and Jenkins Methodology

The Box-Jenkins methodology uses an iterative approach of identifying a possible model from a general class of models. The chosen model is then checked against the historical data to see whether it accurately describes the series. The models fit well if the residuals are generally small, randomly distributed, and contain no useful information. If the specified model is not satisfactory, the process is repeated using a new model designed to improve on the original one. This iterative procedure continues until a satisfactory model is found. At this point, the model can be used for forecasting. The Box and Jenkins Methodology involves Model Identification, Parameter Estimation, Diagnostic Checking, and Forecasting.

Results and Discussion

Data Analyses

Descriptive Statistics Results

Table 1: Descriptive Statistic Results

	EXR	OP
Mean	232.4974	68.16202
Median	157.2850	65.34000
Maximum	1510.329	138.7400
Minimum	98.78000	14.28000
Std. Dev.	186.8859	29.61777
Skewness	3.585356	0.288117
Kurtosis	20.72311	2.102083
Jarque-Bera	4447.256	13.84933
Probability	0.000000	0.000983
Sum	67889.24	19903.31
Sum Sq. Dev.	10163568	255268.8
Observations	292	292

Source: EViews Output

Table 1 presents the descriptive statistic results of the variables under study. The average exchange rate and oil price were 232.4974 Naira per dollar and 68.16202 USD per barrel, respectively. The maximum and minimum exchange rates were 1510.329 and 98.78 Naira per USD. Similarly, the maximum and minimum oil prices in USD per barrel are 138.7400

and 14.28000. The Jarque-Bera Statistic for exchange rate 4447.256 with p-value < 0.05 and oil price 13.84933 with p-value < 0.05 the series were not normally distributed.

Unit Root Test Using ADF and PP

Table 2: ADF and PP Unit Root Result – Exchange Rate in Naira/USD

Method	Difference Order	Test Critical Values			Test Statistic	P-value	Remark
		1%	5%	10%			
ADF	0	-3.9913	-3.4260	-3.1362	2.1053	1.000	Not Stationery
	1	-3.9913	-3.4260	-3.1362	1.7038	1.000	Not Stationery
	2	-3.9914	-3.4261	-3.1362	-3.5809	0.0333	Stationery
PP	0	-3.9900	-3.4253	-3.1358	1.8628	1.000	Not Stationery
	1	-3.9900	-3.4254	-3.1358	-13.1016	0.000	Stationery

Source: Extracted from EVIEWS Output

Table 2 presents the ADF and PP results of the unit root test for the exchange rate in Naira/USD. The ADF results revealed that the exchange rate was not stationary at the level and after the first difference. This is evidenced by the ADF statistic = 2.1053, $p > 0.05$ for level and ADF statistic = 1.7038, $p > 0.05$ for first difference. However, the series was stationary after the second difference (ADF test statistic = -3.5809, $p < 0.05$). The PP results revealed the series to be stationary after the first difference (PP test statistic = -13.1016, $p < 0.05$).

Table 3: ADF and PP Unit Root Result – Oil Price in USD/Barrels

Method	Difference Order	Test Critical Values			Test Statistic	P-value	Remark
		1%	5%	10%			
ADF	0	-3.9900	-3.4254	-3.1358	-2.7925	0.2013	Not Stationery
	1	-3.9906	-3.4257	-3.1360	-7.9272	0.0000	Stationery
PP	0	-3.9899	-3.4253	-3.1358	-2.5316	0.3126	Not Stationery
	1	-3.9900	-3.4254	-3.1358	-12.7807	0.000	Stationery

Source: Extracted from EVIEWS Output

Table 3 presents the ADF and PP results for the unit root test of oil price in USD per barrel. Both ADF and PP results revealed that the series was stationary after the first difference (ADF statistic = -7.9272, $p < 0.05$; PP test statistic = -12.7807, $p < 0.05$).

ARIMA Model Fitting of Exchange Rate Oil Price

The Grid Search method, where the appropriate model is one that is both stationary and invertible, is employed in fitting the ARIMA Model for Exchange Rate and Oil Price. Among the possible ARIMA models applied, the ARIMA (2, 2, 4) for exchange rate and ARIMA (2,1,0) for oil price emerges as the optimal model with the least Akaike Information Criterion (AIC) (Model Selection Criteria Table Appendix B).

Table 4: Result of ARIMA (2,2,4) – Exchange Rate

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.364835	0.828652	0.440276	0.6601
AR(1)	0.037136	0.031128	1.193011	0.2339
AR(2)	-0.929573	0.028514	-32.60103	0.0000
MA(1)	-0.962883	0.039014	-24.68032	0.0000
MA(2)	0.502842	0.033419	15.04656	0.0000
MA(3)	-0.958384	0.036041	-26.59128	0.0000
MA(4)	0.756608	0.026926	28.10006	0.0000
SIGMASQ	995.5812	36.67720	27.14442	0.0000
R-squared	0.610877	Mean dependent var		-0.934945
Adjusted R-squared	0.601218	S.D. dependent var		50.66929
S.E. of regression	31.99726	Akaike info criterion		9.829536
Sum squared resid	288718.6	Schwarz criterion		9.930774
Log-likelihood	-1417.283	Hannan-Quinn criter.		9.870097
F-statistic	63.24371	Durbin-Watson stat		1.963757
Prob(F-statistic)	0.000000			
Inverted AR Roots	.02+.96i	.02-.96i		
Inverted MA Roots	.85-.32i	.85+.32i	-.37+.89i	-.37-.89i

Source: EViews Output

The estimated parameter of ARIMA (2,2,4) for the exchange rate was presented in Table 4. Parameters for AR(2), MA(1), MA(2), MA(3) and MA(4) were statistically significant at all level of significance (p -value = 0.000). However, the constant term and the parameter of AR(1) were not statistically significant (p -value = 0.6601). The estimated model is $Y_t = 0.364835 + 0.037136Y_{t-1} - 0.929573Y_{t-2} - 0.962883X_{t-1} + 0.502842X_{t-2} - 0.958384X_{t-3} + 0.756608X_{t-1} + \varepsilon$. Where Y_t represent the current series.

Table 5: Result of ARIMA (2,1,0) – Oil Price

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003868	0.009127	0.423750	0.6721
AR(1)	0.239353	0.038141	6.275479	0.0000
AR(2)	-0.172126	0.059531	-2.891360	0.0041
SIGMASQ	0.011992	0.000440	27.24470	0.0000
R-squared	0.070266	Mean dependent var		0.003902
Adjusted R-squared	0.060548	S.D. dependent var		0.113768
S.E. of regression	0.110270	Akaike info criterion		-1.557769
Sum squared resid	3.489758	Schwarz criterion		-1.507277
Log-likelihood	230.6555	Hannan-Quinn criter.		-1.537542
F-statistic	7.230178	Durbin-Watson stat		2.013424
Prob(F-statistic)	0.000108			
Inverted AR Roots	.12-.40i	.12+.40i		

Source: EViews Output

The estimated parameter of ARIMA (2,1,0) for the oil price is presented in Table 5. The parameters for AR(1) and AR(2) were statistically significant at all levels of significance (p-value = 0.0041 and 0.0000, respectively). However, the constant term was not statistically significant (p-value = 0.6721). The estimated model is $Y_t = 0.003868 + 0.239353Y_{t-1} - 0.172126Y_{t-2} + \varepsilon$

Residual Diagnostic of Fitted Exchange Rate ARIMA (2,2,4) Model

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1 0.003	0.003	0.0020	
		2 -0.018	-0.018	0.0942	
		3 -0.037	-0.037	0.5038	
		4 -0.089	-0.089	2.8330	
		5 0.092	0.092	5.3551	
		6 0.028	0.023	5.5827	
		7 0.056	0.054	6.5372	0.011
		8 0.098	0.099	9.4273	0.009
		9 -0.119	-0.102	13.687	0.003
		10 -0.063	-0.062	14.900	0.005
		11 -0.048	-0.041	15.594	0.008
		12 0.003	-0.001	15.597	0.016
		13 0.043	0.000	16.161	0.024
		14 0.009	0.010	16.184	0.040
		15 0.029	0.031	16.441	0.058
		16 0.070	0.089	17.943	0.056
		17 0.027	0.065	18.167	0.078
		18 0.003	0.010	18.170	0.111
		19 0.024	0.027	18.347	0.145
		20 -0.009	-0.019	18.370	0.190
		21 -0.041	-0.066	18.904	0.218
		22 -0.005	-0.021	18.913	0.273
		23 -0.002	-0.016	18.915	0.333
		24 -0.026	-0.050	19.137	0.383

Figure 5: Correlogram Q-statistic of residuals of Fitted Exchange Rate ARIMA (2, 2, 4) Model

Figure 5 presents the correlogram Q-statistic of residuals of the fitted ARIMA (2,2,4) model for the exchange rate. The spikes in the ACF and the PACF for the estimated model is relatively small or approximately equal to zero that is, the spikes are within the two-error bound or 95% confidence interval. This suggests that the residuals are white noise implying that the model is adequate. However, some of the p-values for instance at 7 – 14 were less than 0.05. This suggests that the series is volatile in such periods.

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.046	0.046	0.6226	0.430
		2	0.081	0.079	2.5633	0.278
		3	0.111	0.105	6.2309	0.101
		4	0.011	-0.004	6.2666	0.180
		5	0.122	0.107	10.719	0.057
		6	0.010	-0.010	10.751	0.096
		7	0.072	0.057	12.322	0.090
		8	0.166	0.143	20.642	0.008
		9	0.008	-0.011	20.662	0.014
		10	0.010	-0.036	20.691	0.023
		11	-0.001	-0.030	20.692	0.037
		12	-0.001	-0.011	20.692	0.055
		13	-0.001	-0.029	20.692	0.079
		14	-0.000	0.003	20.692	0.110
		15	0.001	-0.013	20.693	0.147
		16	-0.001	-0.020	20.693	0.191
		17	-0.000	0.004	20.693	0.240
		18	0.001	0.013	20.693	0.295
		19	-0.001	0.007	20.694	0.354
		20	-0.001	0.004	20.694	0.415
		21	-0.001	0.005	20.694	0.478
		22	0.001	-0.000	20.695	0.540
		23	0.002	0.003	20.695	0.600
		24	-0.001	0.001	20.696	0.657

Figure 6: Correlogram of residuals Squared of Fitted Exchange Rate ARIMA (2, 2, 4) Model

Figure 6 presents the correlogram of residuals squared of the fitted ARIMA (2,2,4) model for the exchange rate. Also, the spikes in the ACF and the PACF for the estimated model are relatively small or approximately equal to zero that is, the spikes are within the two-error bound or 95% confidence interval. This also suggests that the residuals are white noise implying that the model is adequate. However, series is volatile since some of the p-values were less than 0.05 in lag 8, 9 and 10.

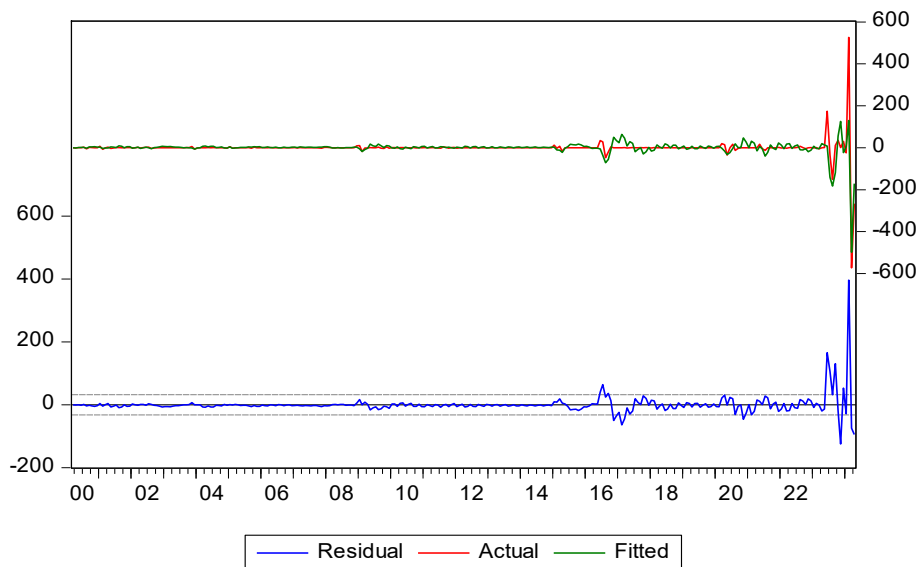


Figure 7: Residual Versus Actual Graph of the fitted Exchange Rate ARIMA (2,2,4) model

Figure 7 revealed some spiky residuals in high volatile periods such as the months of 2023 and 2024. The residual plots are quite similar to the ones for the difference series. Thus, suggesting that the estimated model is adequate.

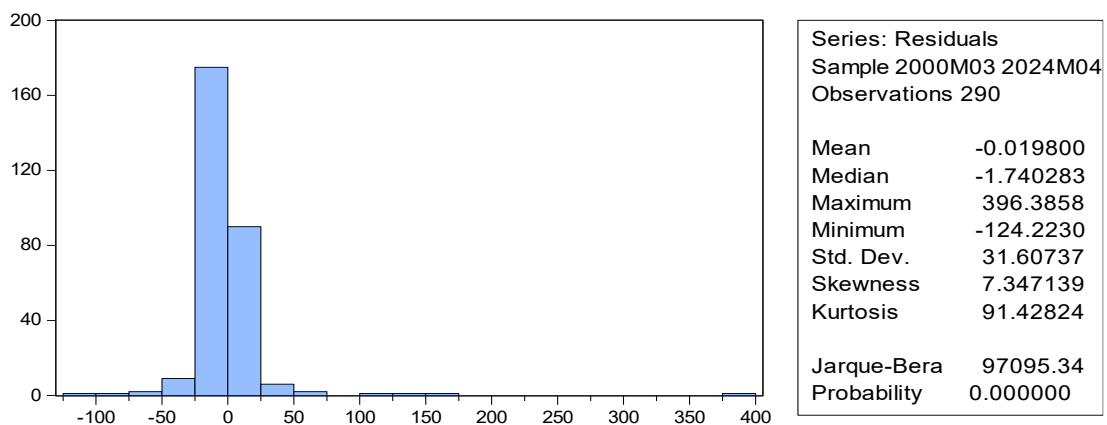


Figure 8: Histogram of residuals of Fitted Exchange rate ARIMA (2,2,4) Model

In Figure 8 presents the histogram of residuals. The mean value of the residuals is -0.01980 is approximately zero. The values of skewness and kurtosis are 7.347139 and 91.42824 respectively. This means that the residuals have excessive kurtosis and slightly skewed to the right. Jarque-Bera test shows that the residuals series is not normally distributed at 5% significance level (Jarque-Bera statistic = 97095.34 , $p < 0.05$).

Residual Diagnostic of Fitted Oil Price ARIMA (2,1,0) Model

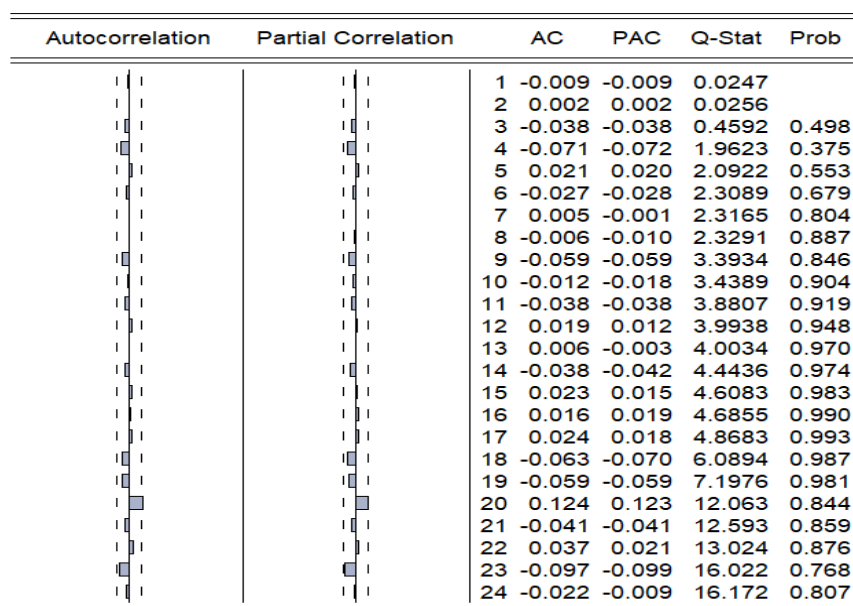


Figure 9: Correlogram Q-statistic of residuals of Fitted Oil Price ARIMA (2, 1, 0) Model

Figure 9 presents the correlogram Q-statistic of residuals of fitted ARIMA (2,1,0) model for oil price. The spikes in the ACF and the PACF for the estimated model is relatively small or approximately equal to zero that is, the spikes are within the two-error bound or 95% confidence interval. This suggesting that the residuals are white noise implying that the model is adequate.

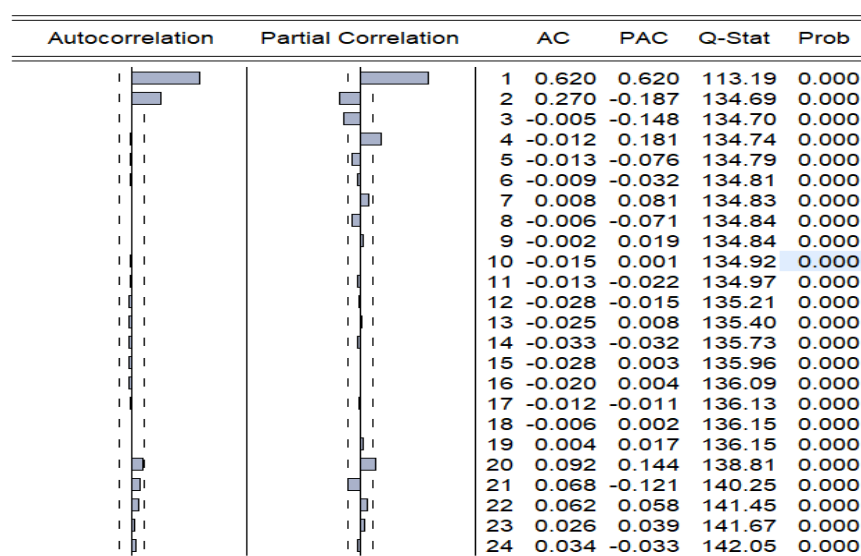


Figure 10: Correlogram of residuals Squared of Fitted Oil Price ARIMA (2, 1, 0) Model

Figure 10 presents the correlogram of residuals squared of the fitted ARIMA (2,1,0) model for the oil price.

The spikes in the ACF and the PACF for the estimated model are relatively small or approximately equal to zero that is, the spikes are within the two-error bound or 95% confidence interval. This suggests that the residuals are white noise implying that the model is adequate.

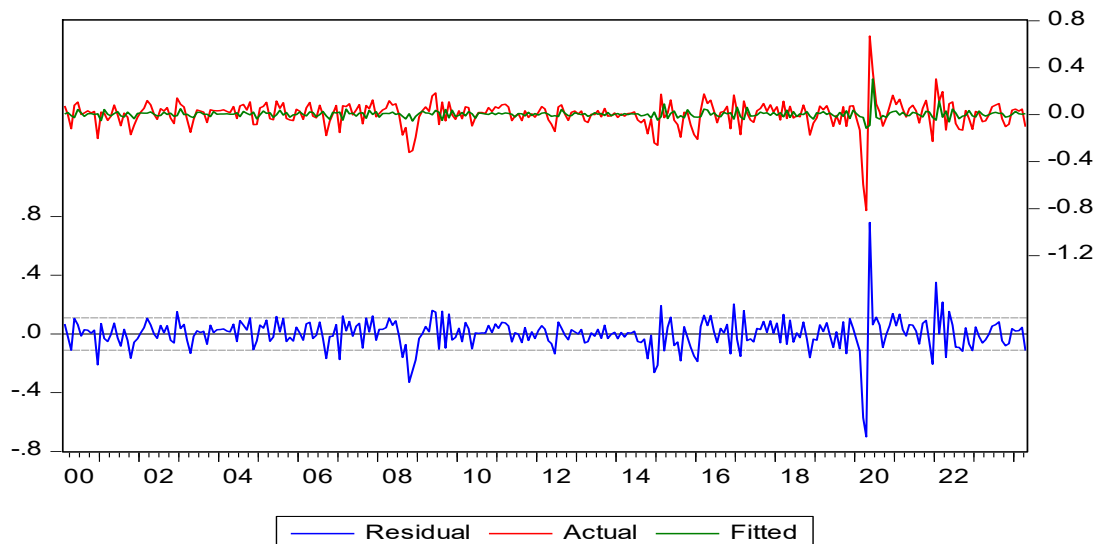


Fig 11: Residual Versus Actual Graph of the fitted Oil Price ARIAM (2,1,o) model

Figure 11 revealed some spiky residuals in highly volatile periods such as 2023 and 2022. The residual plots are quite similar to the ones for the difference series. Thus, suggesting that the estimated model is adequate.

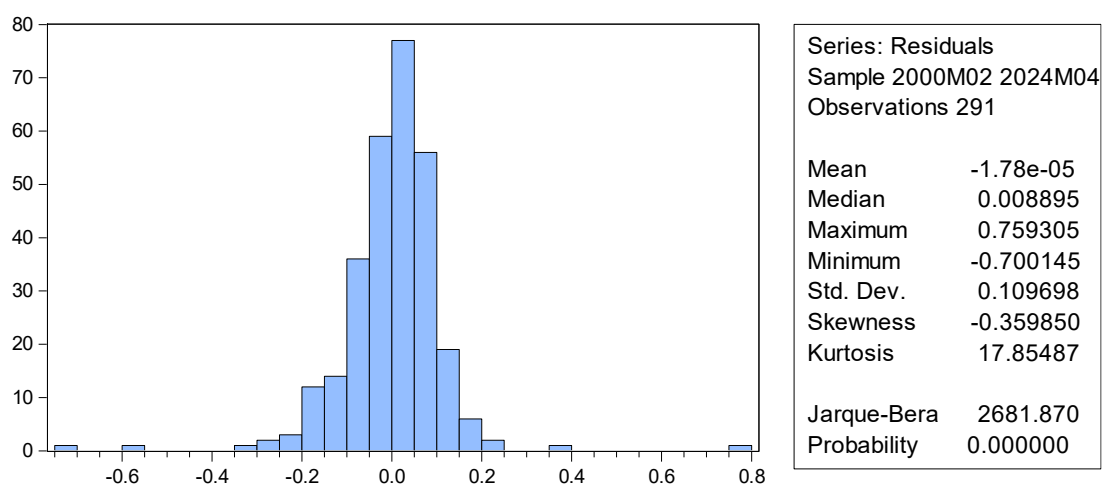


Figure 12: Histogram of residuals of Fitted Oil Price ARIMA (2,1,o) Model

In Figure 12 presents the histogram of residuals of the fitted ARIMA (2,1,o) model. The mean value of the residuals is -0.0000178 which is approximately zero.

The values of skewness and kurtosis are -0.359850 and 17.85487 respectively. This means that the residuals have excessive kurtosis. The Jarque-Bera test shows that the residuals series is not normally distributed at a 5% significance level (Jarque-Bera statistic = 2861.870, $p < 0.05$).

GARCH Model Fitting of Exchange Rate and Oil Price EXRRETURNS

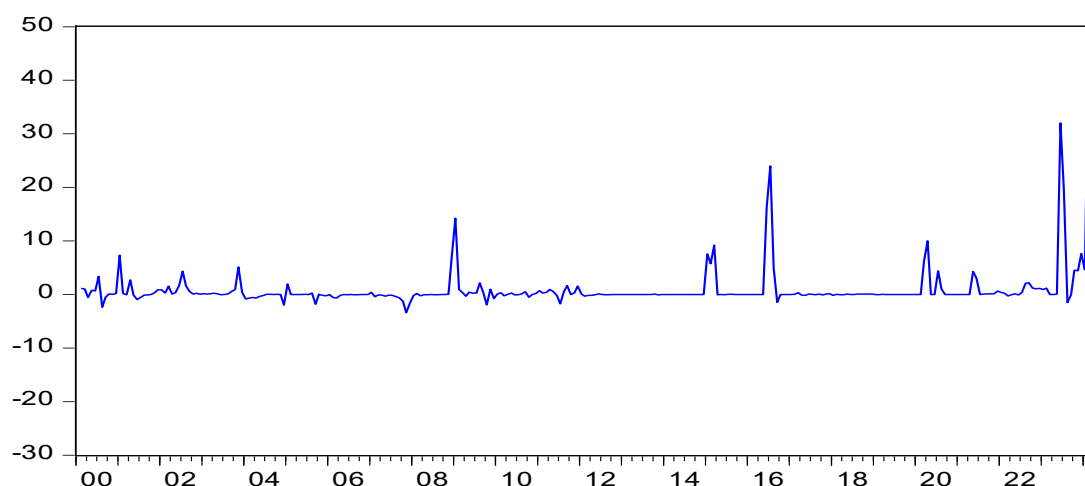


Figure 13: Time Plot of Exchange Rate Returns (EXRRETURNS)

Figure 13 presents the time plot of the exchange rate returns for the study periods. The graph revealed evidence of volatility clustering that is, a period of large changes followed by a period of large changes and vice versa. There was also mean reversal as the volatility always reversed back to zero. Thus, indicating that the series is stationary with volatility clustering and mean reversals.

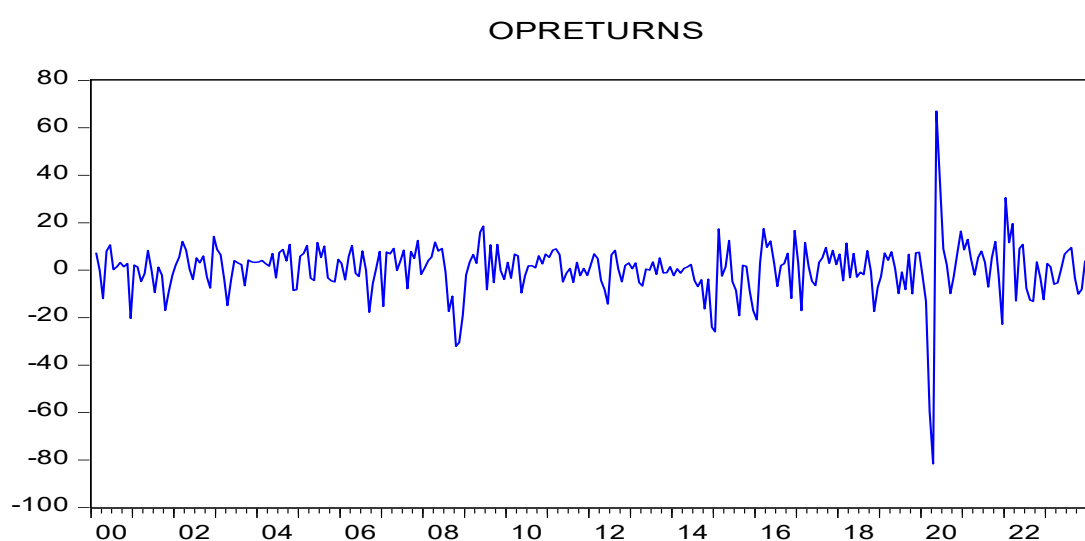


Figure 14: Time Plot of Oil Price Returns (OPRETURNS)

Figure 14 presents the time plot of oil price returns for the study periods. The graph revealed evidence of volatility clustering and mean reversal. This is evidenced by the period of large changes followed by a period of large changes and vice versa. Thus, indicating that the series is stationary with volatility clustering and mean reversals.

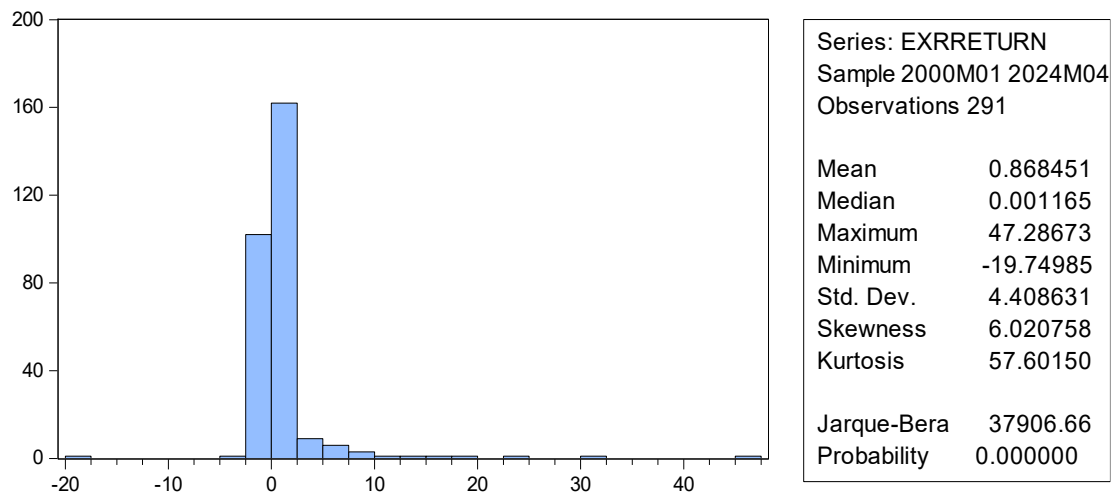


Figure 15: Histogram of Exchange Rate Returns

Figure 15 presents the histogram of exchange rate returns which shows a distribution with most returns near zero but with a significant presence of extreme values, particularly large positive returns. The mean return is slightly positive at 0.868451, while the median is near zero, indicating a balanced spread around zero. The distribution is highly positively skewed (skewness = 6.020758) and has high kurtosis (57.60150). The Jarque-Bera test confirms significant deviation from normality (statistic = 37906.66, p-value = 0.000000). This indicates high volatility.

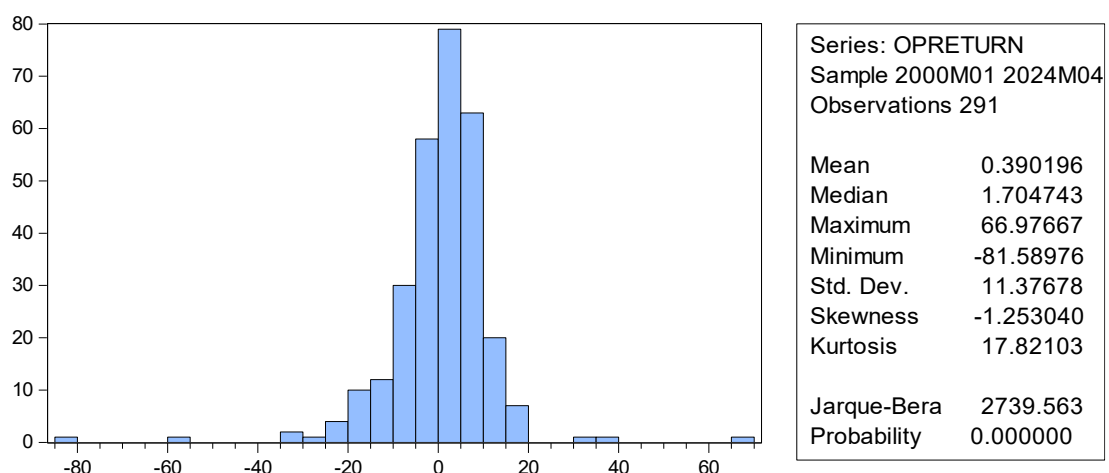


Figure 16: Histogram of Returns on Oil Price

Figure 16 presents the histogram of Returns on Oil Price which shows that most returns are near zero, but there are occasional large negative returns. The average return is slightly

positive at 0.390196, with the median higher at 1.704743, indicating a slight positive skew. The distribution is negatively skewed (skewness = -1.253040) and has high kurtosis (17.82103). The Jarque-Bera statistic is 2739.563, with a probability of 0.000000, implying that the distribution of returns significantly deviates from a normal distribution.

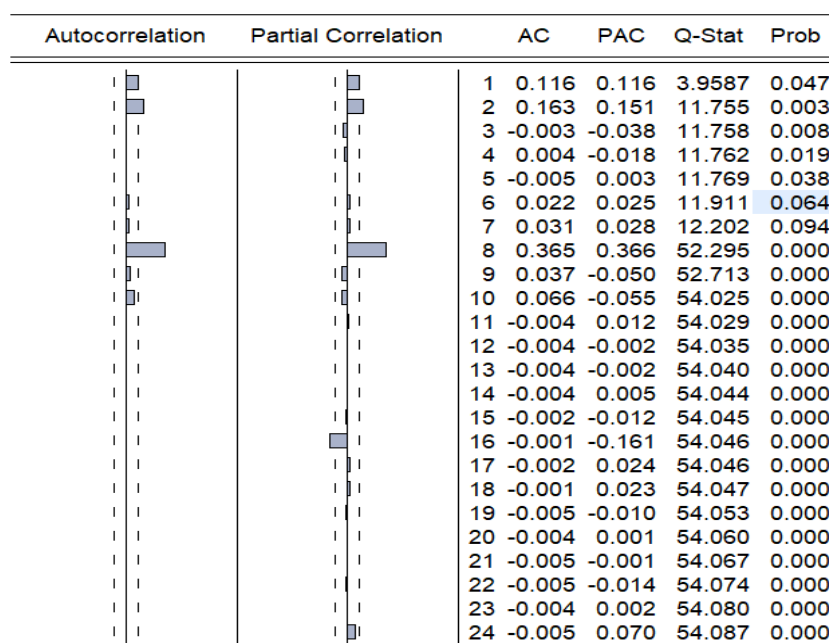


Figure 17: Correlogram of residuals squared of Exchange Rate Returns

Figure 17 presents the correlogram of residuals squared of exchange rate returns. The graph revealed some spike in lag 1, 2, 8 and 16 in the ACF and PACF and all the p-values less than 0.05 indicating that the exchange rate returns are volatile.

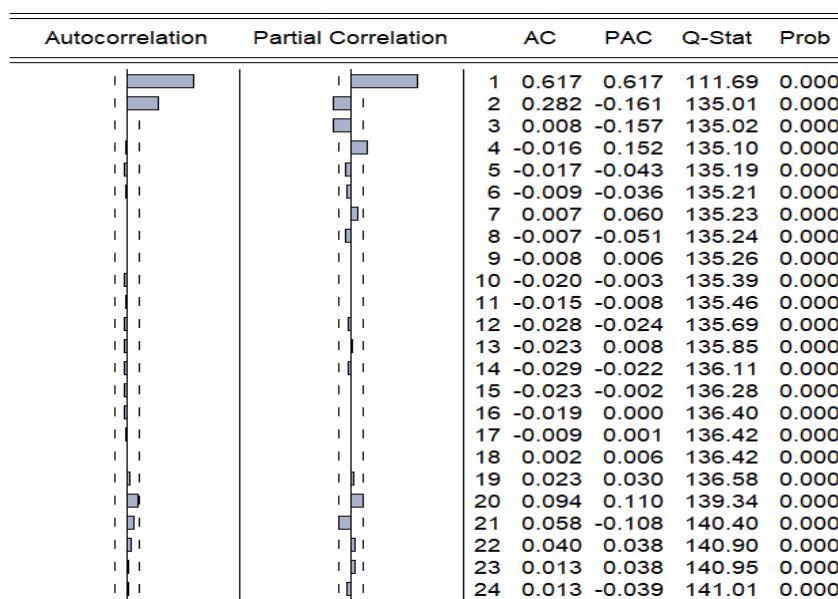


Figure 18: Correlogram of residuals squared of Oil Price Returns

Figure 18 presents the correlogram of residuals squared of oil price returns. The graph revealed spikes in lag 1,2 and 3 in the ACF and PACF and all the p-values were less than 0.05 indicating that the oil price returns are volatile.

Table 6: ARCH LM Test for Exchange Rate Returns

F-statistic	4.045562	Prob. F(1,287)	0.0452
Obs*R-squared	4.017128	Prob. Chi-Square(1)	0.0450

Source: EViews Output

Table 6 revealed F-statistic = 4.045562, Prob(1, 287) = 0.0452 < 0.05 and Obs*R- squared = 4.01728, Prob(chi-square) = 0.0450 < 0.05 indicating that the exchange rate returns series was heteroskedastic indicating that there is presence of ARCH effect. Thus, providing the justification for using GARCH models.

Table 7: ARCH LM Test for Oil Price Return

F-statistic	176.8319	Prob. F(1,287)	0.0000
Obs*R-squared	110.1788	Prob. Chi-Square(1)	0.0000

Source: EViews Output

Table 7 revealed F-statistic = 176.8319, Prob(1, 287) = 0.0000 < 0.05 and Obs*R- squared = 110.1788, Prob(chi-square) = 0.0000 < 0.05 indicating that the oil price returns series was heteroskedastic indicating that there is presence of ARCH effect. Thus, providing the justification for using GARCH models.

Table 8: Model Selection Criterion – Exchange Rate returns

Order of GARCH	Criterion	Error Distributions		
	Used	Normal Dist.	Student's t.	GED
(1,1)	Log-likelihood	-743.8376	-141.5175	-147.2673
	SIC	5.1644	1.0933	1.1329
	AIC	5.1644	1.0174	1.0570
	HQIC	5.1897	1.0479	1.0874
(1,2)	Log-likelihood	-742.9573	-135.0117	-137.4231
	SIC	5.2412	1.0680	1.0886
	AIC	5.1652	0.9794	0.9962
	HQIC	5.1956	1.0149	1.0315
(2,1)	Log-likelihood	-743.3869	-352.6163	-149.4526
	SIC	5.2441	2.5687	1.1676
	AIC	5.1682	2.4801	1.0790
	HQIC	5.1986	2.5156	1.1145

Source: Extracted from EViews Output

From Table 8 it can be observed that the GARCH (1,2) model with student's t error distribution construct has the least value of AIC, SIC, HQIC and a high value of log likelihood. Thus, the GARCH (1,2) model with the student's t-distribution error construct was considered the optimal GARCH model for estimating monthly average official exchange rate returns.

Table 9: Model Selection Criterion – Oil Price returns

Order of GARCH	Criterion	Error Distributions		
		Normal Dist.	Student's t. Dist.	GED
(1,1)	Log-likelihood	-1037.863	<u>-1030.140</u>	-1032.050
	SIC	7.2554	<u>7.2217</u>	7.2349
	AIC	7.1922	<u>7.1458</u>	7.1590
	HQIC	7.2175	<u>7.1762</u>	7.1894
(1,2)	Log-likelihood	-1049.972	-1049.972	-1059.055
	SIC	7.3585	7.3585	7.4407
	AIC	7.2826	7.2826	7.3521
	HQIC	7.3130	7.3130	7.3876
(2,1)	Log-likelihood	-1037.550	-1030.037	-1031.992
	SIC	7.2728	7.2406	7.2540
	AIC	7.1969	7.1520	7.1655
	HQIC	7.2273	7.1875	7.2010

Source: Extracted from EVIEWS Output

From Table 9 it can be observed that the GARCH (1,1) model with student's-t error distribution construct has the least value of AIC, SIC, HQIC and a high value of log-likelihood. Thus, the GARCH (1,1) model with the student's t-distribution error construct was considered the optimal GARCH model for estimating monthly oil price returns.

Table 10: Parameter Estimate of GARCH (1,2) with Student's t Distribution – Exchange Rate Returns

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000114	0.000674	0.169939	0.8651
EXRRETURNS(-1)	0.198819	0.039062	5.089889	0.0000
Variance Equation				
C	0.059224	51.00702	0.001161	0.9991
RESID(-1)^2	15460.63	13319356	0.001161	0.9991
RESID(-2)^2	2516.823	2168291.	0.001161	0.9991
GARCH(-1)	0.002751	0.003363	0.818006	0.4134
T-DIST. DOF	2.000087	0.074546	26.83021	0.0000
R-squared	0.046064	Mean dependent var		0.867523
Adjusted R-squared	0.042751	S.D. dependent var		4.416223
S.E. of regression	4.320793	Akaike info criterion		0.996021
Sum squared residual	5376.744	Schwarz criterion		1.084605
Log-likelihood	-137.4231	Hannan-Quinn criter.		1.031512
Durbin-Watson stat	1.625880			

Source: EViews Output

Table 10 presents the parameter estimates of GARCH (1,2) with students-t error distribution for exchange rate returns. The mean equation is $0.000114 + 0.198819EXRRETURNS(-1)$ implies that the average exchange rate return is 0.000114 and its past value significantly predict the current exchange rate returns (p-value < 0.05) by 0.198819. The variance equation is $\hat{h}_t = 0.059224 + 0.002751\hat{h}_{t-1} + 15460.63\hat{\mu}_{t-1} + 2516.823\hat{\mu}_{t-2}$. The GARCH Coefficient is positive and statistically insignificant at the 5% level. The time-varying volatility includes a constant 0.059224 plus its past $0.002751\hat{h}_{t-1}$ which depends on past errors $15460.63\hat{\mu}_{t-1}$ and $2516.823\hat{\mu}_{t-2}$. The findings clearly establish the presence of time-varying conditional volatility of returns of the exchange rate. The estimated sum of ARCH and GARCH parameters ($0.002751 + 15460.63 + 2516.823 > 1$) is large indicating that the stability conditions were not met. Both ARCH and GARCH coefficients had positive signs implying that the effect of this month's shock remains in the forecast of variance for many periods in the future.

Table 11: Parameter Estimate of GARCH (1,2) with Student's t Distribution – Oil Price Returns

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.051199	0.472437	2.225054	0.0261
OPRETURNS(-1)	0.199162	0.067883	2.933908	0.0033
Variance Equation				
C	29.12220	9.980496	2.917911	0.0035
RESID(-1)^2	0.449031	0.145898	3.077709	0.0021
GARCH(-1)	0.265272	0.156527	1.694733	0.0901
T-DIST. DOF	6.568316	2.306980	2.847149	0.0044
R-squared	0.037032	Mean dependent var		0.366178
Adjusted R-squared	0.033688	S.D. dependent var		11.38905
S.E. of regression	11.19557	Akaike info criterion		7.145794
Sum squared resid	36098.16	Schwarz criterion		7.221722
Log-likelihood	-1030.140	Hannan-Quinn criter.		7.176215
Durbin-Watson stat	1.907519			

Source: EViews Output

Table 11 presents the parameter estimates of GARCH (1,1) with students-t error distribution for oil price returns. The mean equation is $1.051199 + 0.199162OPRETURNS(-1)$ implies that the average oil price return is 1.051199 and its past value significantly predict the current oil price returns ($p - \text{value} < 0.05$) by 0.199162. The variance equation is $\hat{h}_t = 29.12220 + 0.265272\hat{h}_{t-1} + 0.449031\hat{\mu}_{t-1}$. The GARCH coefficient is positive and statistically significant at 10% level.

The time-varying volatility includes a constant 29.12220 plus its past $0.265272\hat{h}_{t-1}$ which depends on past errors $0.449031\hat{\mu}_{t-1}$. The findings clearly establish the presence of time-varying conditional volatility of returns of oil prices. The estimated sum of ARCH and GARCH parameters ($0.449031 + 0.265272 < 1$), indicates that the stability conditions were met. This also indicates that the persistence of volatility shocks is large since is approximately equal to 1. In addition, both coefficients were positive. This implies that the effect of this month's shock remains in the forecast of variance for many periods in the future.

Residuals Diagnostics for Fitted GARCH Models

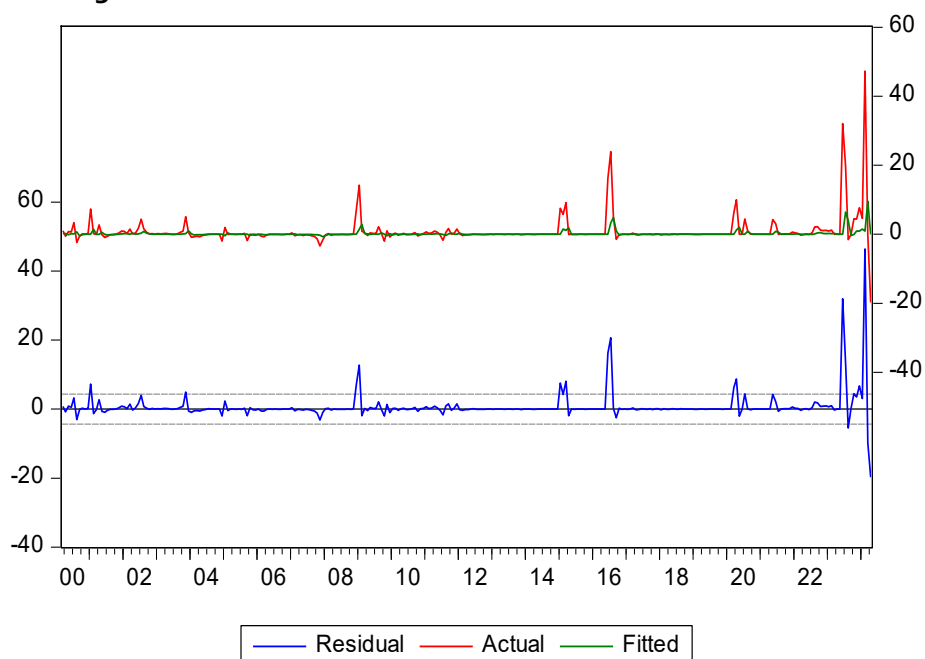


Figure 19: Actual, Fitted Residuals Graph for GARCH (1,2) – Exchange Rate Return

In Figure 19, it was observed that some spiky residuals in high volatile periods such as some months of 2008, 2016, 2020, 2023 and 2024. The residuals plots are quite similar to the one for the exchange rate returns series. Thus, suggesting that the estimated model is adequate.

Autocorrelation		Partial Correlation		AC	PAC	Q-Stat	Prob*	
				1	-0.005	-0.005	0.0080	0.929
				2	-0.005	-0.005	0.0160	0.992
				3	-0.005	-0.005	0.0242	0.999
				4	-0.005	-0.005	0.0324	1.000
				5	-0.005	-0.005	0.0407	1.000
				6	-0.005	-0.005	0.0491	1.000
				7	-0.005	-0.005	0.0575	1.000
				8	-0.005	-0.006	0.0661	1.000
				9	-0.005	-0.006	0.0747	1.000
				10	-0.005	-0.006	0.0835	1.000
				11	-0.005	-0.006	0.0923	1.000
				12	-0.005	-0.006	0.1012	1.000
				13	-0.005	-0.006	0.1102	1.000
				14	-0.005	-0.006	0.1185	1.000
				15	-0.005	-0.006	0.1277	1.000
				16	-0.005	-0.006	0.1370	1.000
				17	-0.005	-0.006	0.1455	1.000
				18	-0.006	-0.006	0.1550	1.000
				19	-0.006	-0.006	0.1646	1.000
				20	-0.005	-0.006	0.1730	1.000
				21	-0.006	-0.006	0.1828	1.000
				22	-0.006	-0.006	0.1926	1.000
				23	-0.006	-0.006	0.2026	1.000
				24	-0.006	-0.006	0.2127	1.000

Figure 20: Correlogram Q-Statistic Residuals for Fitted GARCH (1, 2) – Exchange Rate Returns

Figure 20 present correlogram Q-Statistic residuals for fitted GARCH (1,2) model for exchange rate returns, it can be observed that there is no evidence of serial correlation in the residuals of the estimated model since all the p-values are greater than 0.05 level of significance. Also, all the spikes of ACF and PACF lies within the 95% confidence intervals indicates that the model is good.

Autocorrelation		Partial Correlation		AC	PAC	Q-Stat	Prob*	
				1	-0.004	-0.004	0.0046	0.946
				2	-0.004	-0.004	0.0092	0.995
				3	-0.004	-0.004	0.0139	1.000
				4	-0.004	-0.004	0.0187	1.000
				5	-0.004	-0.004	0.0234	1.000
				6	-0.004	-0.004	0.0283	1.000
				7	-0.004	-0.004	0.0332	1.000
				8	-0.004	-0.004	0.0381	1.000
				9	-0.004	-0.004	0.0431	1.000
				10	-0.004	-0.004	0.0481	1.000
				11	-0.004	-0.004	0.0532	1.000
				12	-0.004	-0.004	0.0583	1.000
				13	-0.004	-0.004	0.0635	1.000
				14	-0.004	-0.004	0.0688	1.000
				15	-0.004	-0.004	0.0741	1.000
				16	-0.004	-0.004	0.0794	1.000
				17	-0.004	-0.004	0.0848	1.000
				18	-0.004	-0.004	0.0903	1.000
				19	-0.004	-0.005	0.0958	1.000
				20	-0.004	-0.005	0.1014	1.000
				21	-0.004	-0.005	0.1070	1.000
				22	-0.004	-0.005	0.1127	1.000
				23	-0.004	-0.005	0.1185	1.000
				24	-0.004	-0.005	0.1243	1.000

Figure 21: Correlogram Squared Residuals for Fitted GARCH (1, 2) – Exchange Rate Returns

Figure 21 presents a correlogram of Squared residuals for the fitted GARCH (1,2) model for exchange rate returns, it can also be observed that there is no evidence of serial correlation in the residuals of the estimated model since all the p-values are greater than 0.05 level of significance and all the spikes of ACF and PACF lies within the 95% confidence intervals indicating that the model is good.

Table 12: Heteroskedasticity Test: ARCH

F-statistic	0.004497	Prob. F(1,287)	0.9466
Obs*R-squared	0.004529	Prob. Chi-Square(1)	0.9463

Source: EViews Output

Table 12 presents the results of heteroscedasticity test in the residuals of the fitted GARCH (1,2) model for exchange rate returns. The F-statistic = 0.004497 with p-value = 0.9466 indicates that there is no evidence of heteroscedasticity in the residuals of the estimated

model. Thus, there is no ARCH effect in the estimated model implying that the model is adequate.

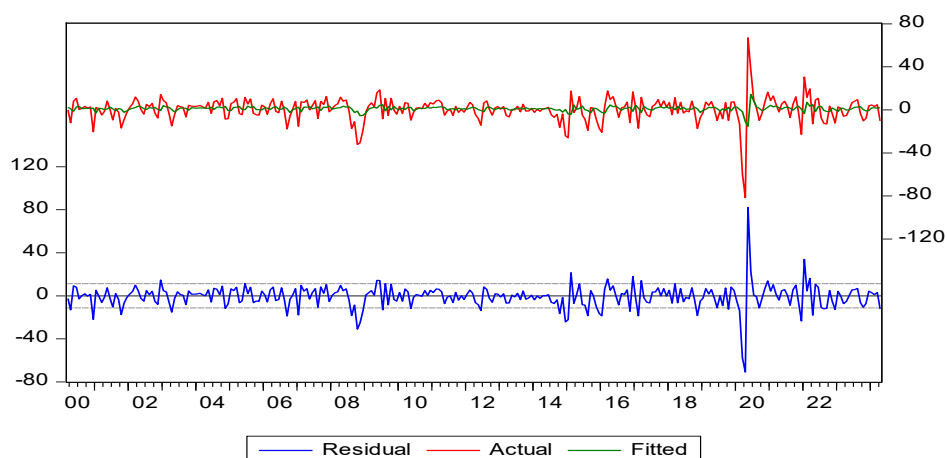


Figure 22: Actual, Fitted Residuals Graph for GARCH (1,1) – Oil Price Returns

In Figure 22, it was observed that some spiky residuals in high volatile periods such as some months of 2020. The residuals plots exhibit similar pattern with actual oil price returns series. Thus, suggesting that the estimated model is adequate.

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*	
		1	0.068	0.068	1.3643	0.243
		2	-0.043	-0.047	1.8977	0.387
		3	-0.049	-0.043	2.5993	0.458
		4	0.003	0.007	2.6012	0.627
		5	0.021	0.017	2.7365	0.741
		6	-0.085	-0.090	4.8945	0.557
		7	0.041	0.057	5.4091	0.610
		8	-0.019	-0.032	5.5204	0.701
		9	-0.088	-0.090	7.8287	0.552
		10	-0.007	0.009	7.8448	0.644
		11	0.044	0.039	8.4352	0.674
		12	0.026	0.000	8.6427	0.733
		13	-0.039	-0.028	9.1185	0.764
		14	-0.056	-0.049	10.090	0.756
		15	0.017	0.009	10.175	0.809
		16	0.042	0.040	10.714	0.827
		17	0.007	-0.000	10.730	0.870
		18	-0.028	-0.032	10.967	0.896
		19	-0.021	-0.016	11.104	0.920
		20	0.029	0.032	11.376	0.936
		21	0.069	0.070	12.893	0.912
		22	0.043	0.031	13.475	0.919
		23	-0.008	-0.018	13.495	0.940
		24	-0.121	-0.114	18.135	0.796

Figure 23: Correlogram Residuals Q-statistics for GARCH (1,1) - oil price Returns

Figure 23 present correlogram Q-Statistic residuals for fitted GARCH (1,1) model for oil price returns, it can be observed that there is no evidence of serial correlation in the residuals of

the estimated GARCH (1,1) model since all the p-values are greater than 0.05 level of significance. Also, all the spikes of ACF and PACF lies within the 95% confidence intervals indicates that the model is good.

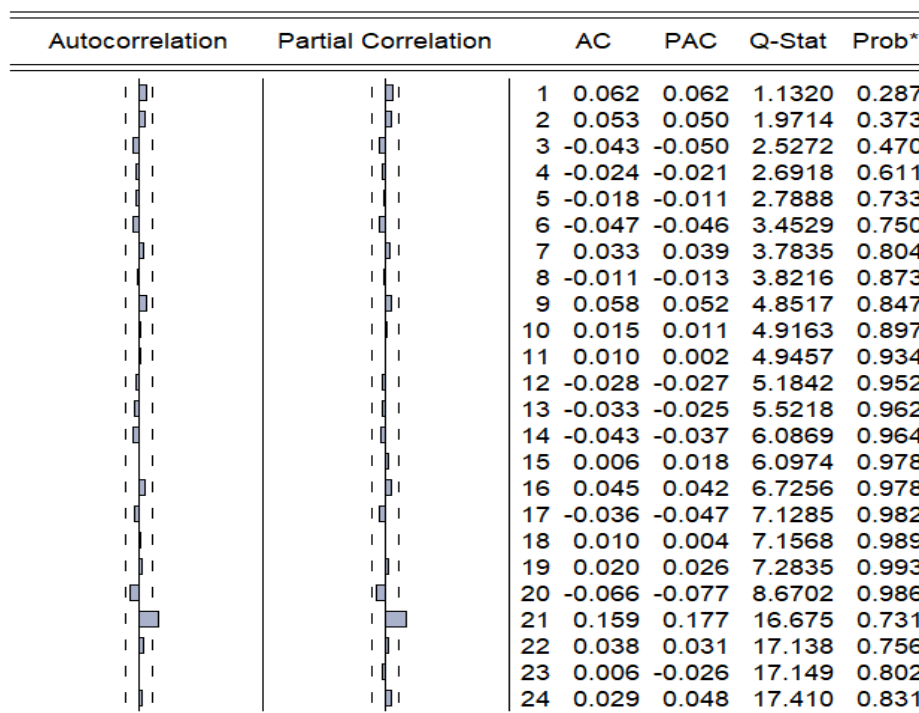


Figure 24: Correlogram Squared Residuals for GARCH (1,1) - oil price Returns

Figure 24 presents correlogram squared residuals for fitted GARCH (1,1) model for oil price returns, it can also be observed that there is no evidence of serial correlation in the residuals of the estimated GARCH (1,1) model since all the p-values are greater than 0.05 level of significance. Also, all the spikes of ACF and PACF lie within the 95% confidence intervals, which indicates that the model is good.

Table 13: Heteroskedasticity Test: ARCH

F-statistic	1.118964	Prob. F(1,287)	0.2910
Obs*R-squared	1.122385	Prob. Chi-Square(1)	0.2894

Source: EViews Output

Table 13 presents the results of the heteroscedasticity test in the residuals of the fitted GARCH (1,1) model for oil price returns. The F-statistic = 1.118964 with p-value = 0.2910 indicates that there is no evidence of heteroscedasticity in the residuals of the estimated model. Thus, there is no ARCH effect in the estimated model, indicating that the model is adequate.

Evaluation of Models Performances

The performance of the fitted models was evaluated in terms of information criterion and forecast performances.

Information Criterion for ARIMA (2,2,4) and GARCH (1,2) Models – Exchange Rate

Table 14: Information criterion for ARIMA (2,2,4) and GARCH (1,2) - t model

Model	AIC	SIC	HQIC	Log-likelihood
ARIMA (2,2,4)	9.8295	9.9308	9.8701	-1417.283
GARCH (1,2) –t	0.9794	1.0680	1.0149	-135.0117

Source: Extracted from E-VIEWS Output

Table 14 presents the information criterion (AIC, SIC and HQIC) obtained from equation estimation from both ARIMA (2, 2, 4) and GARCH (1, 2) with student's t distribution models. It was found that AIC, SIC and HQIC values for GARCH (1,2) with student's t were smaller as compared to that of ARIMA (2, 2, 4) model. Thus, the GARCH (1,2) with student's t performed better than the ARIMA (2,2,4) model in estimating the exchange rate in Nigeria.

Information Criterion for ARIMA (2,1,0) and GARCH (1,1) Models – Oil Price

Table 15: Information criterion for ARIMA (2,1,0) and GARCH (1,1) - t model

Model	AIC	SIC	HQIC
ARIMA (2,1,0)	-1.5578	-1.5073	-1.5375
GARCH (1,1) –t	7.1458	7.2217	7.1762

Source: Extracted from E-VIEWS Output

Table 15 presents the information criterion (AIC, SIC and HQIC) obtained from equation estimation from both ARIMA (2, 1, 0) and GARCH (1, 1) with student's t distribution models. It was found that AIC, SIC and HQIC values for ARIMA (2, 1, 0) were smaller as compared to that of GARCH (1,1) with t-distribution model. Thus, the ARIAM (2,1,0) performed better than the GARCH (1,1) model in estimating the oil price in Nigeria.

Forecast Performance for ARIMA (2,2,4) and GARCH (1,2) Models – Exchange Rate

Table 16: Forecasting Performance for ARIMA (2,2,4) and GARCH (1,2) - t model

Model	RMSE	MAE	MAPE	Theil – U
ARIMA (2,2,4)	31.6645	11.2589	3.5192	0.0531
GARCH (1,2) –t	4.3059	1.2183	NA	0.8039

Source: Extracted from E-VIEWS Output

From Table 16 it can be concluded that all forecast errors from the GARCH (1,2)-t model are smaller than those from the ARIMA (2,2,4) model. Therefore, it was concluded that the GARCH (1,2)-t model performs better than ARIMA (2, 2, 4). In other words, GARCH (1, 2)-t is a better forecast model for monthly exchange rate than ARIMA (2,2,4) model.

Forecast Performance for ARIMA (2,1,0) and GARCH (1,1) Models – Oil Price

Table 17: Forecast Performance for ARIMA (2,1,0) and GARCH (1,1) - t model

Model	RMSE	MAE	MAPE	Theil – U
ARIMA (2,1,0)	6.3909	4.5368	7.2956	0.0428
GARCH (1,1) –t	11.1569	7.1145	202.8260	0.8022

Source: Extracted from E-VIEWS Output

From Table 17, it can be concluded that all forecast errors from the ARIMA (2,1,0) model are smaller than those from the GARCH (1,1)-t model. Therefore, it was concluded that the ARIMA (2,1,0) model performs better than GARCH (1,1)-t.

Discussion of Findings

The time plot of the exchange rate revealed a positive trend for the whole period of study while that of oil price fluctuated over time. The descriptive statistic revealed that both series (exchange rate and oil price) were not normally distributed (Jarque-Bera statistic = 4447.256, $p < 0.05$; Jarque-Bera statistic = 13.84933, $p\text{-value} < 0.05$) for exchange rate and oil price respectively. Using the ADF test, it was discovered that the exchange rate was stationary after the second difference, while the oil price was stationary after the first difference. In the case of the PP test, both series were stationary after the first difference. Among the possible ARIMA models applied, the ARIMA (2, 2, 4) for exchange rate and ARIMA (2,1,0) for oil price emerges as the optimal model with the least Akaike Information Criterion (AIC) (Model Selection Criteria Table Appendix B). The diagnostic test using correlogram Q-statistic and correlogram residuals squared revealed some spikes and some p -values less than 0.05, especially in the ARIMA (2,2,4), indicating that the exchange rate was volatile. However, the residual diagnostics for the ARIMA (2,1,0) model were adequate. The ARCH LM test for both exchange rate and oil price revealed the presence of ARCH Effects $F\text{-statistic} = 4.045562$, $\text{Prob} (1, 287) = 0.0452 < 0.05$ and $F\text{-statistic} = 176.8319$, $\text{Prob} (1, 287) = 0.0000 < 0.05$. Thus, providing a justification for generalized autoregressive conditional heteroscedasticity modelling to take care of the stylised property of the series. To fit the appropriate GARCH model to the financial times series data, the returns series was computed for series, and the time plot of the returns series indicates the presence of volatility clustering and mean reversals, which are basic properties of financial time series data. The ARCH LM test of the returns series revealed the presence of the ARCH effect ($F\text{-statistic} = 4.045562$, $\text{Prob} (1, 287) = 0.0452 < 0.05$ and $F\text{-statistic} = 176.8319$, $\text{Prob} (1, 287) =$

0.0000 < 0.05) for exchange rate and oil price returns, respectively. Using AIC, SIC, HQIC and Log-likelihood as model selection criteria, it was discovered that the GARCH (1,2) model with the student's t-distribution error construct and GARCH (1,1) model with the student's t-distribution error construct were appropriate for modelling the exchange rate and oil price returns.

The residuals diagnostic using correlogram Q-statistics and correlogram of residuals squared revealed that the models were adequate as all the spikes were within the 95% confidence intervals. Also, the ARCH LM test revealed the absence of the ARCH effect (F-statistic = 0.004497 with p-value = 0.9466 and F-statistic = 1.118964 with p-value = 0.2910) for exchange rate returns and oil price returns, respectively.

The performance evaluation of the models revealed that GARCH (1,2) with students' t-distribution performance was better than ARIMA (2,2,4) for modelling and forecasting monthly exchange rates in Nigeria. Because the GARCH(1,2)-t model achieved a lower Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE) compared to ARIMA, indicating higher forecasting accuracy. Additionally, the GARCH model's residuals exhibited no significant autocorrelation or heteroskedasticity, confirming its ability to capture the underlying volatility structure of the data. In contrast, the ARIMA model, while effective for capturing linear trends and dependencies, failed to account for the time-varying volatility and heavy-tailed distributions inherent in exchange rate data. This limitation likely contributed to its inferior performance, particularly during periods of high market uncertainty or economic instability. The GARCH(1,2)-t model's ability to model volatility clustering, fat-tailed distributions, and asymmetric effects makes it a more suitable choice for forecasting exchange rates in Nigeria. Also, it was discovered that ARIMA (2,1,0) performs better than GARCH (1,1) with students' t distributions in modelling and forecasting monthly oil prices in Nigeria.

Conclusions

The study investigates the performance of some alternative models (ARIMA and GARCH) in modelling and forecasting crude oil prices and exchange rates in Nigeria. Based on the findings of the study, it was concluded that GARCH (1,2)-t and ARIMA (2,1,0) are the optimal models suitable for modelling and forecasting exchange rates and oil prices, respectively. This may be due to some reason like high volatility in exchange rate data, which is rare in oil prices. The findings of this study have significant implications for Nigeria's economic policy, particularly in managing exchange rate volatility and oil price fluctuations. Nigeria's heavy reliance on oil exports makes its economy highly susceptible to global oil price shocks, which directly impact foreign exchange earnings and, consequently, the exchange rate. For policymakers, these results suggest that traditional linear models like ARIMA may not adequately capture the complexities of exchange rate behavior in Nigeria. Instead, models that explicitly account for volatility, such as GARCH, should be prioritized for exchange rate forecasting. Accurate exchange rate forecasts are critical for the Central Bank of Nigeria (CBN) in formulating effective monetary policies,

managing foreign reserves, and stabilizing the Naira. Furthermore, the study's findings on oil price forecasting highlight the need for Nigeria to diversify its economy and reduce its dependence on oil revenues. Accurate oil price forecasts can guide the government in budgeting and planning, especially during periods of price instability.

Recommendations

Based on the findings of the study, the following recommendations were made;

- i. Policy makers may wish to use the GARCH (1, 2)-t for forecasting feature values of exchange rate returns in Nigeria. This may assist in informed decision-making.
- ii. Also, policymakers may wish to use ARIMA (2,1,0) for forecasting future values of oil prices in Nigeria.
- iii. Oil price is among the financial time series data that may exhibit volatility and are suitably modelled using GARCH; however, it was found that the ARIMA model outperformed GARCH. Thus, future researchers may wish to compare other classes of GARCH models with ARIMA in modelling oil price data in Nigeria.

Data Availability

The data used for the study was obtained from the Central Bank of Nigeria (CBN) Website and the United States Energy Administration. The Data for oil prices are available at the United States Energy Administration with the URL (<https://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=INl0000008&f=M>), and the data for Exchange Rates are available at the CBN Website.

Conflict of Interest

There is no conflict of interest.

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