

Mathematics as a Geoheat Engine to Determine the Lithospheric Thermal Properties from an Aeromagnetic Data Over Southeastern Taraba State

Samson Oyine Ankeli¹; Ademayowa Isaac Adejumobi²; Ahmed Idowu Agbelejoye³; Oluwafemi Victor Bamikole¹; Musa Ayinde Tijani⁴; Kunle Barnabas Oladapo¹ and Jeffrey Ilabeshi⁵

¹Federal University of Technology Akure, Akure, Nigeria. ²University of Benin, Benin City, Nigeria. ³Osun State University, Osogbo, Nigeria. ⁴King Saudi University, Saudi Arabia. ⁵Federal Polytechnic Auchi, Auchi, Nigeria.

Corresponding author: ankeli.samson@yahoo.com

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Abstract

The study is based on mathematical-geophysical model that incorporates potential field theory, Fourier spectral analysis and lithospheric thermal modelling as a solid foundation for estimating Curie Point Depth (CPD), geothermal gradient, and surface heat flow from aeromagnetic anomalies of Taraba State for geothermal applications. Aeromagnetic measurements provide indirect constraints on the crustal thermal structure of the earth, but these measurements demand stringent mathematical transformation for meaningful interpretation. The spectral decomposition theory, radial power spectrum analysis and the exponential decay of magnetic energy with depth are the basis of the methodology. FFT-based spectral analysis is a mathematically robust geophysical tool, as it balances between electromagnetism, Thermodynamics, Signal processing and allows to estimate indirectly Curie depth and Geothermal gradient and Heat flow. This mathematical model shows that FFT converts aeromagnetic data into physically meaningful spectral energy, CPD range of 13.16-29.09km was accurately estimated using spectral slope analysis, Geothermal gradient of 19.94-44.08°C/km and heat flow of 49.85-110.19mWm⁻² are mathematical derivatives of CPD and the method is a powerful, non-invasive geothermal exploration tool. The bridge between magnetic signals and the thermal structure of the Earth is mathematics (FFT and spectral theory). Aeromagnetic data without mathematics are noise but with FFT they become a geothermal diagnostic tool.

Keywords: Fast Fourier Transform, Mathematics, Thermal Geophysics, Northeastern Nigeria, Spectral Analysis.

Introduction

Aeromagnetic surveys measure the changes in magnetic field of the earth caused by subsurface, raw data are spatially complex and noisy (Ankeli et al., 2026). Mathematics particularly Fourier analysis offers an interface between raw geophysical signals and

thermal structure of the subsurface. Aeromagnetic anomalies are spatial variations in the magnetic field of the earth due to underlying magnetized rocks. These anomalies are however, multi-scale, non-linear and depth-dependent (Ankeli et al., 2026).

Mathematics especially Fourier theory offers a transformation tool that transforms spatial geophysical signals into frequency domain representations that can be used to estimate depth of magnetic sources. Recent geophysical reports affirm that: FFT-based spectral analysis is popular in geothermal exploration of tectonic environments (Musa et al, 2025), FFT spectral methods are dependable in estimating CPD, geothermal gradient, and heat flow in Nigerian (Nneji et al, 2025), Spectral centroid and radial power spectrum methods are common techniques in crustal thermal investigations (Ike et al., 2024). Aeromagnetic FFT inversion was used globally for Antarctica geothermal modelling (Dziadek et al., 2021). Spectral analysis of aeromagnetic data has been utilized by geoscientists to ascertain the Curie Point Depth (CPD) and investigate the thermal structure of the lithosphere in sedimentary basins and tectonically active regions globally. The method was refined by Tanaka et al., (1999) in the studies of Spector and Grant (1970). It was proven useful in rift scenarios where tectonic deformation and magmatic processes significantly influence heat transfer beneath the deep crust. This method enables scientists to pinpoint areas with significant geothermal potential and enhances their comprehension of heat transmission mechanisms in these regions. This method has been effectively used in a number of studies across Africa to delineate geothermal systems and map out variations in crustal temperature distribution. An example, CPD research discovered remarkably shallow Curie depths in continuous magmatism and extreme geothermal gradients within the East African Rift System (Ebinger et al., 2017). According to empirical data, geothermal resources are strongly associated with locations where the heat flow is high and the CPD is shallow (Anyadiegwu et al., 2022). Studies revealed that geothermal anomalies are often structurally affected by faults and magmatic intrusions (Yohanna et al., 2023). Applications of the same approach have provided valuable information about lithospheric thermal structure and geothermal potential in basins like the Ethiopian Rift, Sudan, and Chad by aeromagnetic spectral analysis (A. Salem et al., 2014). All said and said, these examples prove that aeromagnetic data is highly beneficial in regional geothermal studies, particularly in those cases when deep boreholes data and other underground measurements are difficult to get. In complicated tectonic settings, combined spectrum approaches greatly improve geothermal predictions, according to recent studies (Ezennwa et al., 2026).

Theoretical Framework

Potential field theory, Fourier spectral analysis, and lithospheric thermal modeling are all integrated into this study's mathematical–geophysical framework, which provides a solid foundation for analyzing aeromagnetic data for geothermal applications.

Magnetic Anomalies and Potential Field Theory

Aeromagnetic data represents variations in the Earth's magnetic field due to subsurface magnetization contrast. These fields are appropriate for spectral decomposition since they show linear superposition and satisfy Laplace's equation. Lithological contrasts, structural trends (fractures, faults), and the depth distribution of magnetic bodies are all encoded by magnetic anomalies. Anomaly wavelength features are used to indirectly infer the depth to magnetic sources.

Fourier Transform and Spectral Representation

The Fast Fourier Transform converts spatial-domain magnetic signals into the frequency domain, enabling quantitative depth estimation. High-frequency components are attributed to shallow sources and low-frequency components are attributed to deep sources. This relationship is fundamental because magnetic field amplitudes decay exponentially with depth, allowing spectral slopes to be interpreted in terms of source geometry.

Spectral Depth Estimation Theory

The radially averaged power spectrum is used to estimate depths of magnetic sources. The slope of the logarithmic power spectrum is proportional to depth and multi-layer models allow separation of shallow and deep sources. Recent studies confirm that spectral methods provide reliable depth estimates for geothermal modelling when applied to high-resolution aeromagnetic datasets (Mouzong et al., 2023).

Curie Point Depth (CPD) Theory

The Curie Point Depth is the depth at which magnetic minerals lose ferromagnetism this occur at temperature approximately 580°C. It defines the lower boundary of magnetic sources. This is estimated using centroid method, depth to top and depth to Centroid. CPD reflects the thermal structure of the crust and is widely used as a proxy for geothermal potential (Akar, 2024).

Geothermal Gradient and Heat Flow Modelling

Thermal parameters are derived from CPD assuming conductive heat transfer, Curie temperature (~580°C), heat flow and thermal conductivity. Empirical studies show that

Shallow CPD signifies high geothermal gradient and heat flow and Deep CPD signifies low geothermal activity (Ayigun, 2024).

Integrated Mathematical-Geophysical Framework

The framework of this study combines FFT-based transformation, Spectral slope interpretation, CPD estimation and Thermal parameter modelling. Recent integrated approaches demonstrate that combining spectral and structural analysis improves geothermal predictions in tectonically complex regions (Enzennwa et al., 2026).

Literature Review

Developments of FFT-Based Spectral Analysis.

Recent studies highlight that FFT is one of the fundamental computing tools in the aeromagnetic interpretation, this allows the effective processing of extensive datasets, improved resolution of subsurface structures and allows radially averaged spectral methods. Spectral analysis is still a commonly used technique to estimate Curie depth and geothermal parameters all over the world (Marcelin et al., 2023).

Curie Depth and Heat Flow Experiments.

Recent research (2023-2024) demonstrates that CPD can be estimated reliably from an aeromagnetic data, Strong correlation between CPD and crustal thermal regime and application in geothermal exploration and tectonic studies. Example, spectral analysis was proven to be effective towards the understanding of thermal structures in Cameroon (Mouzong et al., 2023), and Turkey study attributed CPD variations to seismicity and lithospheric structure (Akar, 2024).

Nigerian Studies and Basement Complex.

Recent Nigerian research (2024-2026) points out, significant geothermal variability across the Benue trough tectonic dominance on thermal structure and spectral method applicability in data-limited areas. The most important findings are; CPD varies between 11-35km in the Lower Benue Trough, Geothermal gradients respond between 26-68°C/km, and Heat flow reaches up to 205mW/m² in tectonically active areas (Ezennwa et al., 2026). Likewise, Shallow CPD (less than 10 km) in studies done in Southwestern Nigeria has been associated with magmatic intrusions and geothermal potential (Ike et al., 2024).

Methodological Developments

Recent advances are: Overlapping spectral window methods, combination with structural mapping and utilization of high-resolution NGSA data. Nonetheless, it still has difficulties

that include sensitivity to noises and size of a window, random magnetization assumption and lack of validation by borehole data. Such shortcomings drive the necessity of a sophisticated mathematical model, as suggested in this paper.

Research Gap

Although advances have been made, there are few studies that integrates FFT, spectral theory and thermal modelling into a single mathematical framework, limited geothermal evaluation of Taraba State and in specific and inadequate connectivity between tectonics and thermal parameters. This paper fills these gaps, and offers a quantitative, reproducible model of geothermal assessment.

Research Objectives

The objective of this research is to;

1. develop a mathematical geophysical frame work using FFT base spectral analysis,
2. to convert aeromagnetic data of Taraba State to frequency domain,
3. compute radially averaged power spectra to estimate depths,
4. identify depths to magnetic sources (top and centroid),
5. estimate Curie Point Depth throughout the study area,
6. derives geothermal gradient based on the values of CPD,
7. compute subsurface heat flow distribution,
8. map spatial variation of geothermal parameters within a section of Taraba state,
9. and to relate thermal structure to regional.

Problem Statement

Although the need to utilize renewable energy resources in Nigeria is ever-growing, geothermal energy has not been fully explored as there is limited subsurface temperature information and the cost of direct exploration is high. Aeromagnetic records can be used to investigate the thermal structures of the subsurface indirectly and effectively. Currently, the research often remains sporadic and lacking a unified methodology, with investigations often limited to descriptive interpretations without mathematical integration and giving little attention to the regions of Taraba State, despite its tectonic complexity. Recent studies in the Lower Benue Trough highlights inadequate integration of spectral and structural analysis, despite identification of high geothermal gradients and heat flow in northeastern Nigeria, they are still localized and under-generalized (Yohanna et al., 2023). Hence, there exists a dire need to have a well-rounded mathematical-geophysical model, which would combine the FFT-based spectral analysis with thermal modelling to enhance the accuracy, repeatability and applicability of the geothermal parameter estimation in Taraba State.

Importance of the Research.

This research offers low-cost geothermal exploration technique that improves knowledge crustal thermal structure and illustrates interdisciplinary strength of mathematics in geophysics and assists renewable energy exploration plans.

Study Area Description and Geology

The research region (figure 1a) spans the local governments of Kurmi, Gashaka, and Saduna and is located in Taraba state between latitudes $7^{\circ} 18'$ and $7^{\circ} 36'$ and longitudes $11^{\circ} 0'$ and $11^{\circ} 18'$. Alluvium, Bima sandstone, and the Fika formation are found in the research region, although phyroblastic, magmatic, banded gneiss, and granite predominate there. More than two thirds of the plateau is made up of basement complex rocks that age from the Precambrian to the early Paleozoic era, according to Tukur, Adebayo, & Galtima (2005). The remainder of the plateau, however, is composed of volcanic rocks from the upper Cenozoic to tertiary and quaternary eras, according to Jeje (1983). These rocks are volcanic in nature, extending from cracks, tectonic lines, etc. within the plateau. They are volcanic rocks, which run along the cracks, tectonic lines, etc. on the plateau. These volcanic rocks which comprise the olivine basalt, the basalt mixture and trachyte basalt were observed to contain amphibole and pyroxene combinations as well as small number of other free quartz minerals (Mould, 1960). The tertiary basalts of the Mambilla Plateau are mainly made of common basalts and trachytic lavas (Dupreez and Barber, 1965).

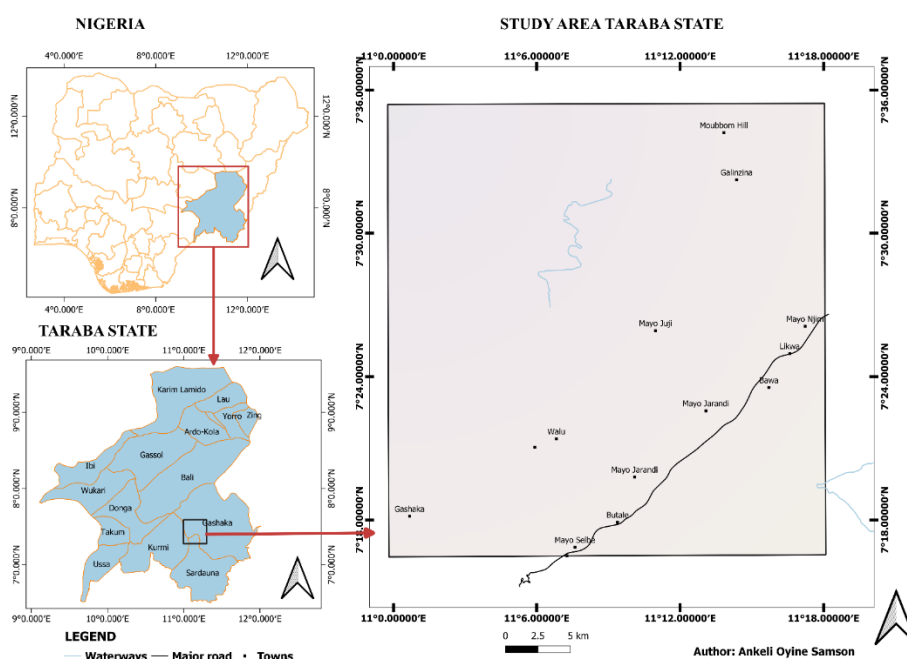


Figure 1a: Map of the study area

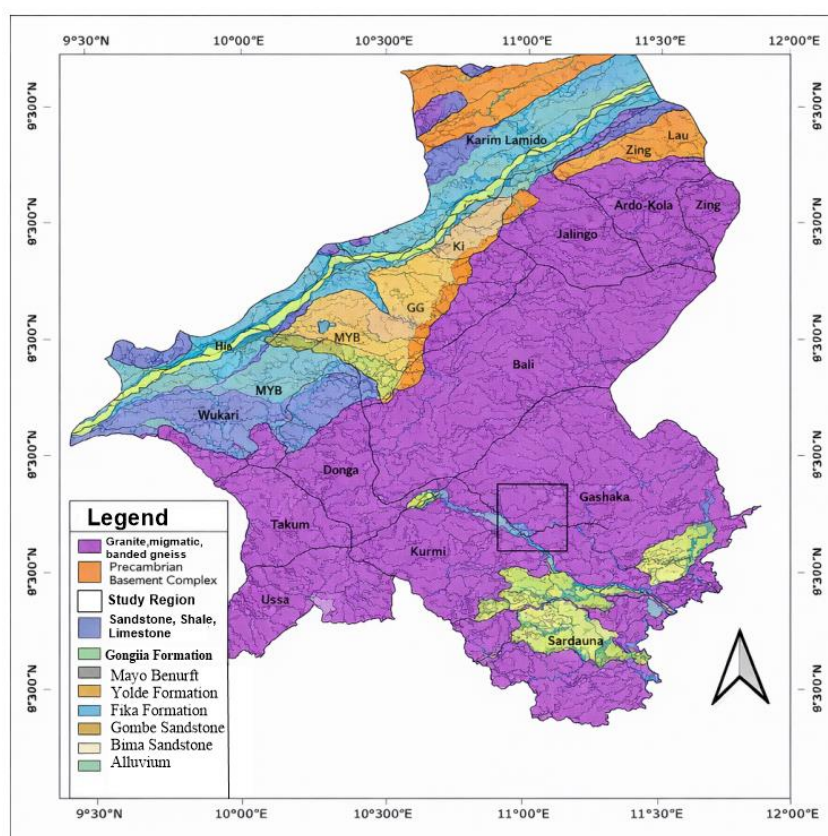


Figure 1b: Geological Map of Taraba State (modelled from NGSA).

Data & Methods

The Nigeria Geological Survey Agency (NGSA) provided aeromagnetic data for the research area. Fugro Airborne Surveys collected the data utilizing a flight configuration of 80 m altitude, 100 m terrain clearance, 100 m line spacing, and 500 m tie-line spacing. The minimum curvature method (Briggs, 1974) was used to create a smooth and continuous surface because the survey data were widely spaced. Using Oasis Montaj software, total magnetic intensity was created. By highlighting changes in magnetic intensity, which are impacted by elements including magnetic susceptibility, depth, and the type of underlying geological structures, the TMI map offers initial insights into subsurface characteristics (Obiora et al., 2016). Using a first order polynomial fitted to the data using the least squares approach, the regional anomaly (figure 3) was distinguished from the residual anomaly (figure 2).

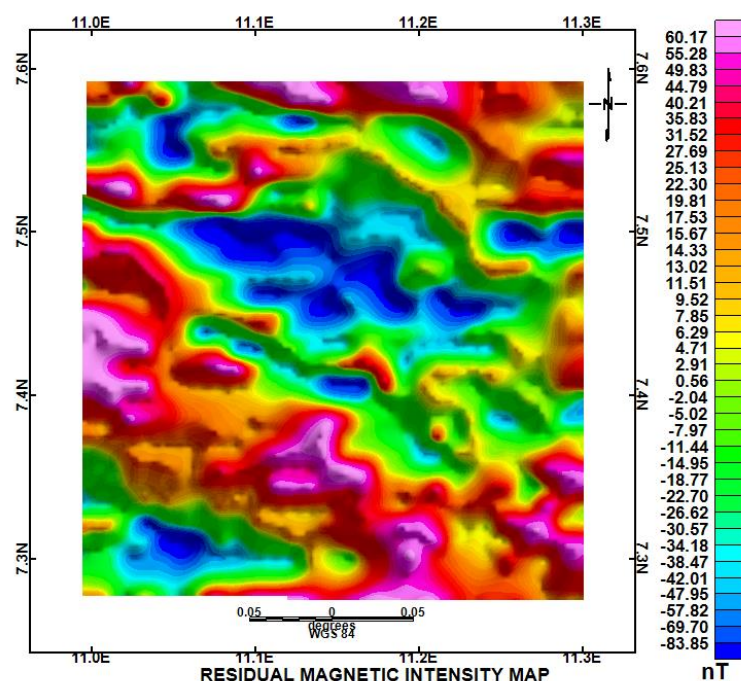


Figure 2: Residual Map of the study area (computation from NGSA aeromagnetic data)

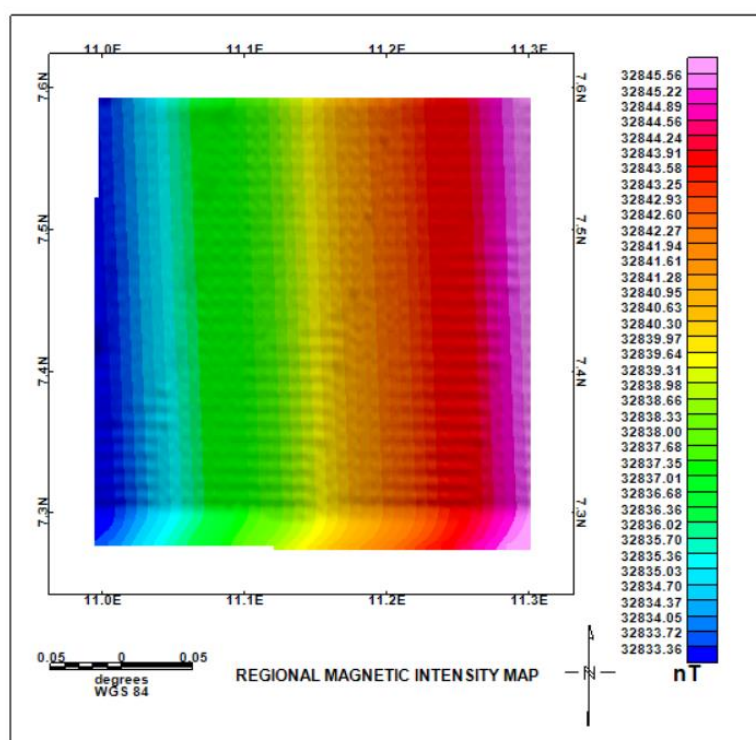


Figure 3: Regional Map of the study area (computation from NGSA aeromagnetic data)

Spectral Analysis

The spectral analysis method is the foundation of the approach utilized in this paper to estimate the Curie surface depth. According to Tanaka et al., (1999) and Okubo et al., (1985) the layer is infinitely long in every horizontal direction. The magnetization $M(x, y)$ is a random function of x and y , and the depth to the top bound of a magnetic source is modest in relation to its horizontal scale. Blakely (1996) presented the total-field anomaly's power-density spectra.

Transformation theory (FFT), statistical spectral analysis, exponential decay modeling, and linear regression in log-power spectra are mathematical techniques that translate geophysical measurements into physically significant parameters. Large aeromagnetic grid processing is made possible, spectral resolution is improved, and computational noise and time are decreased. When dealing with static or quasi-static fields, magnetic surveys simplify Maxwell's equations to

$$\nabla \cdot B = 0 \text{ -----(1)}$$

$$\nabla \times H = j \text{ -----(2) a potential magnetic field exists in a region where there is no electric current therefore } j = 0$$

$$\nabla \times H = 0 \text{ -----(3)}$$

The Relationship between Magnetic flux density B and magnetic field strength H and magnetization M can be expressed in units of field intensity, both numerically and dimensionally, between the two systems, because of the different defining equations for H
 $B = \mu H = (1 + 4\pi\chi)H = H + 4\pi H$ where $(\mu = 1 + 4\pi\chi)$ and $(M = \chi H)$ $B = H + 4\pi M$ in emu

In S.I unit $\mu = \mu_0(1 + \chi)$ From $B = \mu H = \mu_0(1 + \chi)H = \mu_0(H + \chi H) \therefore B = \mu_0(H + M)$ where μ_0 is the magnetic permeability of free space. In region above the magnetic source there are no magnetization $M = 0$, $B = \mu_0 H$ The existence of a scalar magnetic potential from maxwell's equation 2

$$\nabla \times H = 0 \text{ a vector field with zero curl can be written as the gradient of a scalar potential } H = -\nabla\phi, B = \mu_0(-\nabla\phi), \nabla \cdot B = \nabla \cdot (-\mu_0\nabla\phi), \nabla \cdot B = -\nabla^2\phi \text{ since } \mu_0 \text{ is a constant } 0 = -\nabla^2\phi$$

$$\nabla^2\phi = 0 \text{ -----(4)}$$

Above the magnetic sources the magnetic field is source free and can be written as a scalar potential field $B = -\nabla\phi$ since there are no sources above the observation plain. Expanding the Laplace equation $\nabla^2\phi = 0$

$$\nabla^2\phi(x, y, z) = 0 \text{ in Cartesian coordinate;}$$

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

We define the 3D Fourier transform of $\phi(x, y, z)$ as: $\phi(k_x, k_y, k_z) = \iiint_{-\infty}^{\infty} \phi(x, y, z) e^{-i(k_x x + k_y y + k_z z)} dx dy dz$

if we apply the 2-D Fast Fourier transform to a potential field x, y in horizontal direction

$$\begin{aligned}\phi(x, y, z) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(k_x, k_y, z) e^{i(k_x x + k_y y)} dx dy, \quad \phi(k_x, k_y, z) \\ &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(x, y, z) e^{-i(k_x x + k_y y)} dk_x dk_y\end{aligned}$$

Transform the derivatives

$$\mathcal{F} \left[\frac{\partial^2 \phi}{\partial x^2} \right] = -k_x^2 \phi(k_x, k_y, z), \quad \mathcal{F} \left[\frac{\partial^2 \phi}{\partial y^2} \right] = -k_y^2 \phi(k_x, k_y, z), \quad \mathcal{F} \left[\frac{\partial^2 \phi}{\partial z^2} \right] =$$

$\frac{\partial^2 \phi}{\partial z^2}$ (since z is not transformed)

$$\left[-(k_x^2 + k_y^2) \phi + \frac{\partial^2 \phi}{\partial z^2} \right] (k_x, k_y, z) = 0 \quad \text{Defining the radial wave number } K = \sqrt{k_x^2 + k_y^2}$$

$$\begin{aligned}\frac{\partial^2 \phi}{\partial z^2} - K^2 \phi &= 0 \\ \phi''(z) + k^2 \phi(z) &= 0\end{aligned}$$

To solve the second order independent ODE

$$\phi_1(z) = e^{kz} \quad \text{and} \quad \phi_2(z) = e^{-kz},$$

Using Wronskian $W(z)$ of ϕ_1, ϕ_2 is

$$\begin{aligned}W(z) &= \begin{vmatrix} \phi_1 & \phi_2 \\ \phi_1' & \phi_2' \end{vmatrix} \\ W(z) &= \phi_1 \phi_2' - \phi_1' \phi_2 = e^{kz} \cdot (-ke^{-kz}) - e^{-kz} \cdot (ke^{kz}) = -k(e^{kz-kz}) - k(e^{-kz+kz}) \\ &= -2k\end{aligned}$$

Where W is a constant and $k \neq 0$

Using method of variation of parameter

$$\begin{aligned}\phi(z) &= u_1(z) \phi_1(z) + u_2(z) \phi_2(z) \\ \phi(z) &= u_1(z) e^{kz} + u_2(z) e^{-kz}\end{aligned}$$

where $u_1(x)$ and $u_2(x)$ are functions to be found. To simplify derivatives, we impose

$$u_1'(z) e^{kz} + u_2'(z) e^{-kz} = 0 \quad \dots\dots\dots (5)$$

(This is the usual constraint in variation of parameters that removes second derivatives of u_1, u_2 from ϕ'' .)

$$\phi'(z).$$

$$\phi'(z) = u_1'(z) e^{kz} + u_1(z) k e^{kz} + u_2'(z) e^{-kz} - u_2(z) k e^{-kz}$$

Using (A) the terms with u_1' and u_2' cancel, so

$$\begin{aligned}\phi'(z) &= u_1(z) k e^{kz} - u_2(z) k e^{-kz} \\ \phi''(z) &= u_1'(z) k e^{kz} + u_1(z) k^2 e^{kz} - u_2'(z) k e^{-kz} + u_2(z) k^2 e^{-kz} \\ k^2 \phi &= k^2 [u_1(z) e^{kz} + u_2(z) e^{-kz}] = u_1(z) k^2 e^{kz} + u_2(z) k^2 e^{-kz}\end{aligned}$$

Substituting into the ODE

$$\phi'' - k^2 \phi = 0$$

$$\begin{aligned}\phi'' - k^2\phi &= u_1'(z)ke^{kz} + u_1(z)k^2e^{kz} - u_2'(z)ke^{-kz} + u_2(z)k^2e^{-kz} - (u_1(z)k^2e^{kz} + \\ &u_2(z)k^2e^{-kz}) \\ &= (u_1'(z)ke^{kz} - u_2'(z)ke^{-kz}) + u_1(z)k^2e^{kz} - u_1(z)k^2e^{kz} + u_2(z)k^2e^{-kz} - \\ &u_2(z)k^2e^{-kz} \\ &= u_1'(z)ke^{kz} - u_2'(z)ke^{-kz} \\ \phi'' - k^2\phi &= k(u_1'(z)e^{kz} - u_2'(z)e^{-kz})\end{aligned}$$

Our ODE requires this to be zero, so

$$k(u_1'(z)e^{kz} - u_2'(z)e^{-kz}) = 0$$

If $k \neq 0$ divide by k to get the second constraint

$$u_1'(z)e^{kz} - u_2'(z)e^{-kz} = 0 \dots\dots\dots (6)$$

Combining equation (5) and (6) to solve for u_1' and u_2' Solve this 2×2 system (Cramer's rule) to obtain the standard formulas

$$\begin{cases} u_1'e^{kz} + u_2'e^{-kz} = 0 \\ u_1'e^{kz} - u_2'e^{-kz} = 0 \end{cases} \rightarrow \begin{cases} u_1'y_1 + u_2'y_2 = 0 \\ u_1'y_1 - u_2'y_2 = f(z) \end{cases}$$

but in standard variation-of-parameters usage we consider the form

$$\phi'' - k^2\phi = f(z)$$

with $f(z) = 0$ for the homogeneous case. So here $f(z) = 0$.

$$u_1'(z) = -\frac{y_2(z)f(z)}{W(z)} = -\frac{e^{-kz} \cdot 0}{-2k} = 0, \quad u_2'(z) = \frac{y_1(z)f(z)}{W(z)} = \frac{e^{kz} \cdot 0}{-2k} = 0$$

We integrate to get u_1, u_2

$$u_1(z) = -\int u_1'(z) dz = 0, \quad u_1(z) = C_1, \quad u_2(z) = \int u_2'(z) dz = 0, \quad u_2(z) = C_2$$

This is the general solution for $k \neq 0$. (If $k = 0$ the equation reduces to $\phi'' = 0$ and the solution is $\phi(z) = A + Bz$)

$$\phi(z) = C_1e^{kz} + C_2e^{-kz}$$

The total magnetic field T is a combination of the main core geomagnetic field T_0 and the magnetic anomaly due to crustal source ΔT .

$$T = |B|, \quad T = |B_0 + b| = |u_0(H + M)| = T_0 + \Delta T \quad \text{recall } B = u_0(H + M)$$

where B_0 is the earth's main field and b is the anomaly field, because the anomaly field is much lesser than the main field $|b| \ll |B_0|$ the total field anomaly can be approximated as the projection of the anomalous field onto the main field direction. $\Delta T \approx \hat{B}_0 \cdot b$ $\hat{B}_0$ is the unit vector in the direction of the main field and $b = -u_0 \nabla \phi$ Laplace equation for the total magnetic anomaly $\Delta T = -u_0 \hat{B}_0 \cdot \nabla \phi$

Apply Laplacian $\nabla^2 \Delta T = -u_0 \hat{B}_0 \cdot \nabla (\nabla^2 \phi)$ because ϕ satisfies Laplace equation:

$\nabla^2 \phi = 0$ and the Laplacian commutes with derivatives:

$\nabla^2 \Delta T = 0$ since the total magnetic anomaly is a potential field satisfying the Laplace equation above the source, we can express the magnetic anomaly in space domain

produced by a distribution of magnetized sources at a depth. $\nabla^2 \Delta T(x, y, z) = 0$ let the magnetic anomaly be $\Delta T(x, y)$

(Tanaka et al., 1999; Blakely, 1996; Bhattacharyya, 1964) The total-field anomaly is measured at discrete locations, and $\mathcal{F}[\Delta T]$ and θ_f are easily calculated, assume that the layer extends infinitely far in all horizontal directions and that $M(x, y)$ is a random function of x and y . In this case, $\mathcal{F}[\Delta T]$ and $\mathcal{F}[M]$ can be used to express how magnetization behaves statistically. Consider the total-field anomaly measured on a horizontal surface and caused by a horizontal layer with top at depth z_t and thickness z_0 . The total magnetic anomaly in Fourier domain is given by

$$\mathcal{F}[\Delta T] = C \theta_m \theta_f k \tilde{M}(k) e^{-kz}$$

A magnetic layer is bounded by top $z = z_t$ and bottom $z = z_b$ and extends infinitely in the x and y direction

$$\mathcal{F}[\Delta T] = C \theta_m \theta_f k \tilde{M}(k) (e^{-|k|z_t} - e^{-|k|z_b}) = C \theta_m \theta_f k \tilde{M}(k) e^{-|k|z_t} (1 - e^{-|k|(z_b - z_t)})$$

$$\mathcal{F}[\Delta T] = 2\pi C_m m'' \theta'_m \theta'_f e^{-|k|z_t} (1 - e^{-|k|z_t}) = \mathcal{F}[M] \{2\pi C_m \theta_m \theta_f e^{-|k|z_t} (1 - e^{-|k|z_t})\}$$

$$\phi_{\Delta T}(k_x, k_y) = \phi_M(k_x, k_y) \times F(k_x, k_y) \text{-----}(7)$$

where $\phi_{\Delta T}$ and ϕ_M are power-density spectra of the total-field anomaly and the magnetization, respectively, and

$$\Phi_{\Delta T}(k) = |\mathcal{F}[\Delta T]|^2 = C_m^2 |\theta_m|^2 |\theta_f|^2 k^2 |\tilde{M}(k)|^2 e^{-2|k|z_t} (1 - e^{-|k|(z_b - z_t)})^2$$

$$F(k_x, k_y) = 4\pi^2 C_m^2 |\theta_m|^2 |\theta_f|^2 e^{-2|k|z_t} (1 - e^{-|k|(z_b - z_t)})^2$$

$F(k_x, k_y)$ can be analyzed in terms of z_b and z_t . This analysis can be simplified greatly by noting that all terms, except $|\theta_m|^2$ and $|\theta_f|^2$, in equation (7) are radially symmetric. The radial averages of θ_m and θ_f are constants.

$$\Phi_M(k_x, k_y) = |\tilde{M}(k_x, k_y)|^2 = \text{constant} \quad \text{also}$$

$|\theta_m|^2, |\theta_f|^2$, and k^2 are radially symmetric

Hence, the radial average of $\phi_{\Delta T}$ is

$$\bar{\phi}_{\Delta T}(|k|) = A \phi_M(|k|) e^{-2|k|z_t} (1 - e^{-|k|z_t})^2 \text{-----}(8)$$

where A is a constant that depends on the orientations of magnetization and regional field.

We now need a suitable statistical model for $M(x, y)$. For example, if $M(x, y)$ is completely random and uncorrelated, $\phi_M(k_x, k_y)$ is a constant, and equation (8) becomes

$$B = C_m^2 |\theta_m|^2 |\theta_f|^2 k^2 \phi_M, \quad \bar{\phi}_{\Delta T}(|k|) = B e^{-2|k|z_t} (1 - e^{-|k|(z_b - z_t)})^2 \text{ Taking the square root of both sides}$$

$$\sqrt{\bar{\phi}_{\Delta T}(|k|)} = \sqrt{B e^{-2|k|z_t} (1 - e^{-|k|(z_b - z_t)})^2} = \sqrt{B} \cdot e^{-|k|z_t} (1 - e^{-|k|(z_b - z_t)})$$

where B is a constant. Finally, taking the logarithm of both side yields

$$\begin{aligned}\log \left[\sqrt{\bar{\Phi}_{\Delta T}(|k|)} \right] &= \log [B e^{-|k|z_t} (1 - e^{-|k|(z_b - z_t)})] \\ &= \log B + \log e^{-|k|z_t} + \log (1 - e^{-|k|(z_b - z_t)}) \\ &= \log B - |k|z_t + \log(1 - e^{-|k|(z_b - z_t)})\end{aligned}$$

At high wavenumber k which approximate to depth to top layer ($e^{-|k|(z_b - z_t)} \ll 1$ so

$$\log(1 - e^{-|k|(z_b - z_t)}) \approx 0 \text{ therefore } \log \left[\sqrt{\bar{\Phi}_{\Delta T}(|k|)} \right] \approx \log B - |k|z_t \text{ -----(9)}$$

Thus, the depth of the top boundary z_t can be estimated by fitting a straight line through the low wavenumbers of the power spectrum of $\log \left[\sqrt{\bar{\Phi}_{\Delta T}(|k|)} \right]$ on the other hand, equation (8) can be simplified to

$$\bar{\Phi}_{\Delta T}(|k|) = B e^{-2|k|z_0} (e^{-|k|(z_t - z_0)} - e^{-|k|(z_b - z_0)})^2$$

where z_0 is the depth to the central point of the magnetic source. At low wavenumber band $e^{-|k|(z_t - z_0)} = e^{|k|(z_b - z_0)} \approx |k|(z_b - z_0)$ and taking the square root the equation becomes

$$[\Phi_{\Delta T}(|k|)]^{\frac{1}{2}} \approx B \cdot e^{-|k|z_0} (e^{|k|z_0} - e^{-|k|z_0}) = B \cdot e^{-|k|z_0} \cdot (2|k|z_0)$$

where B is a constant and $2z_0$ is the thickness of the total magnetic source, taking the log of both sides

$$\log[\Phi_{\Delta T}(|k|)]^{\frac{1}{2}} = \log[B e^{-|k|z_0}] = \log B + \log e^{-|k|z_0}$$

$$\log \left[\frac{\sqrt{\bar{\Phi}_{\Delta T}(|k|)}}{|k|} \right] = \log B - |k|z_0 \text{ -----(10)}$$

where B is a constant. We could estimate the top bound and the centroid of the magnetic source by fitting a straight line through the high-wavenumber and low-wavenumber parts

of the radially averaged spectrum of $\log \left[\frac{\{\bar{\Phi}_{\Delta T}|k|\}^{\frac{1}{2}}}{|k|} \right]$ and $\log \left[\{\bar{\Phi}_{\Delta T}|k|\}^{\frac{1}{2}} \right]$ from Equation (10)

and (9), respectively. Figure 4b shows an example of a power spectrum of magnetic anomaly data. From the slope of the power spectrum, the top bound and the centroid of a magnetic layer composed of a horizontal (equivalent) layer are estimated. The basal depth of the magnetic source is: $z_b = 2z_0 - z_t$

It is assumed that the Curie point depth is the obtained basal depth of the magnetic source. The average value of the area is represented by the obtained Curie point depth. There is a sensitivity to finite data length, meaning that magnetic anomalies that are only partially represented in the data may deteriorate the predicted spectrum. This is a drawback of the spectrum analysis. The benefit of the spectrum analysis of a magnetic anomaly is the ability to generate estimates of the centroid and top bound of a magnetic source using straightforward assumptions about the magnetization vector and the dimensions of magnetic sources (Blakely, 1996). $\sim 1^\circ \times 1^\circ$ with 50% overlap the spectral window size and percentage overlap used to define the 25 blocks in Table 1.

The $\log \left[\frac{\{\bar{\phi}_{\Delta T}|k|\}^{\frac{1}{2}}}{|k|} \right]$ and $\log \left[\{\bar{\phi}_{\Delta T}|k|\}^{\frac{1}{2}} \right]$ for each square using 2-D FFT power spectrum are calculated. The progress of estimating the Curie point depth of a window is shown in table 1. Finally, the CPD in the study area is calculated after interpolating.

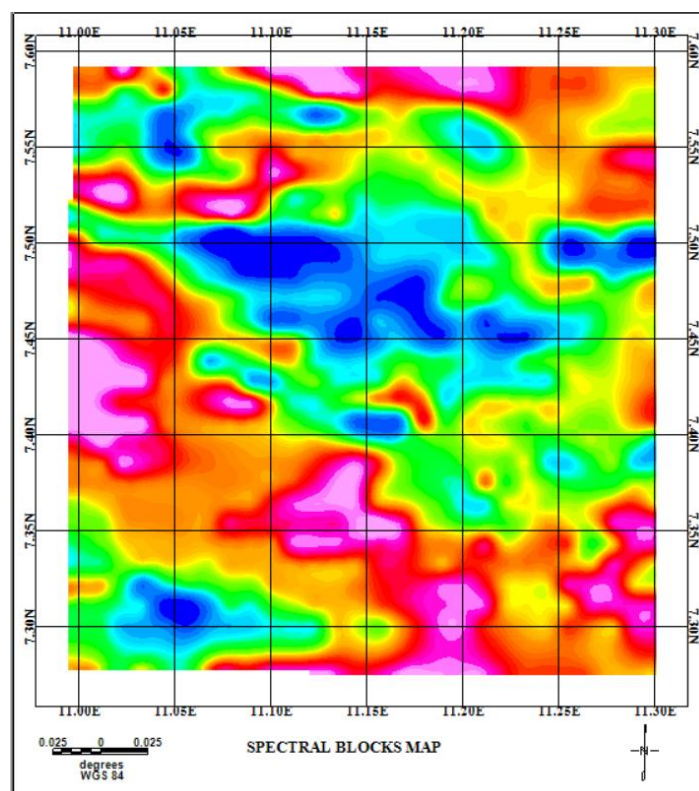
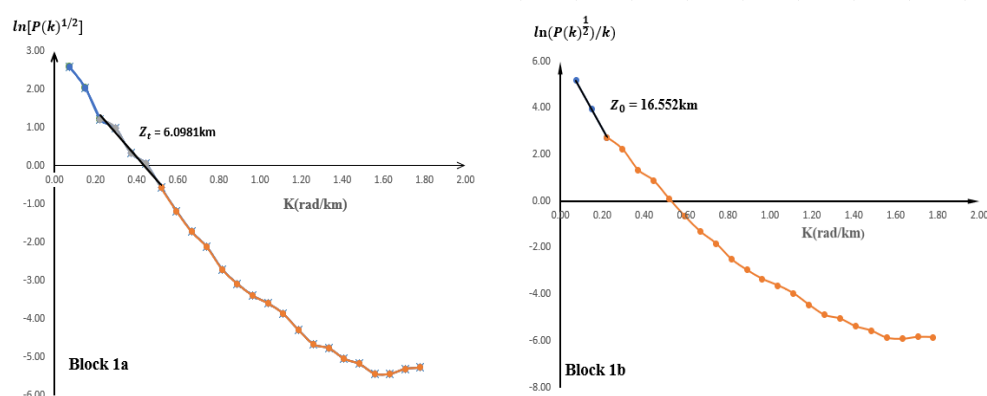


Figure 4a: Spectral blocks map of the research region (computation from NGSA aeromagnetic data)



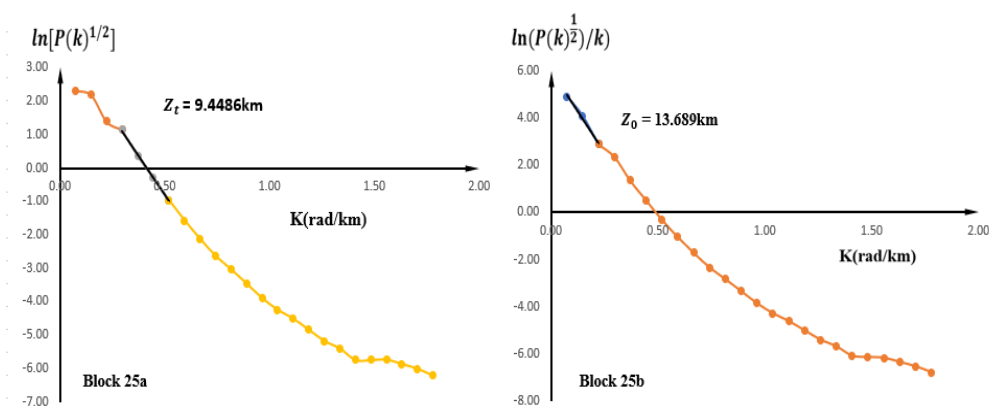


Figure 4b: Spectral Plots for block 1 & 25 (computation from NGSA aeromagnetic data)

Heatflow and Geothermal Gradient Estimation

The governing equation for steady state heat condition assuming 1-D vertical heat conditions in the lithosphere reasonable for curie depth studies, the steady state heat conduction equation is $\frac{d}{dz} \left(k \frac{dT}{dz} \right) + A = 0$ where T is temperature, z is depth, K is thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$) assumed constant, A is radiogenic heat production (Wm^{-3}). Since k is a constant

$$K \frac{d^2 T}{dz^2} + A = 0, \quad \frac{d^2 T}{dz^2} = -\frac{A}{K}, \quad \int \frac{d^2 T}{dz^2} = \int -\frac{A}{K}$$

we have $\frac{dT}{dz} = -\frac{A}{K}z + C_1$ We can define heat flow $q(z)$ from Fourier's law $q(z) = -k \frac{dT}{dz}$

$q(z) = -k \left(-\frac{A}{K}z + C_1 \right)$, $q(z) = Az - kC_1$ The heat flow q_0 at the surface where $z = 0$ is expressed as $q_0 = -kC_1$ and $C_1 = \frac{-q_0}{K}$ so the depth dependent heat flow is $q(z) = q_0 - Az$

If we substitute C_1 into the temperature gradient $\frac{dT}{dz} = -\frac{A}{K}z + C_1$ we have

$$\frac{dT}{dz} = -\frac{A}{K}z - \frac{q_0}{k}$$

$$\int \frac{dT}{dz} = \int \left(-\frac{A}{K}z - \frac{q_0}{k} \right), \quad T(z) = -\frac{A}{2K}z^2 + \frac{q_0}{k}z + C_2$$

At the surface the depth $z = 0$ the temperature T_0 at this depth is given by

$T(0) = -\frac{A}{2K}z^2 - \frac{q_0}{k}z + T_0$ this is the lithospheric geotherm. The Geothermal Gradient $G(z)$ is given by

$G(z) = \frac{dT}{dz} = \frac{q_0}{k} - \frac{A}{K}z$ At shallow depth $A \approx 0$ then $G(z) = \frac{q_0}{k}$ this is the linear geothermal gradient used in many curie studies Curie depth z_b which is defined as the depth at which the temperature reaches the curie temperature T_c . If $T(z_b) = T_c$ For magnetite $T_c = 580^\circ\text{C}$ Substituting into the temperature equation

$T_c = -\frac{A}{2k}z_b^2 + \frac{q_0}{k}z_b + T_0$, $\frac{A}{2k}z_b^2 - \frac{q_0}{k}z_b + (T_c - T_0) = 0$ This is the quadratic equation in curie depth

$$Z_c = \frac{-(-\frac{q_0}{k}) \pm \sqrt{(-\frac{q_0}{k})^2 - 4\frac{A}{2k}(T_c - T_0)}}{2(\frac{A}{2k})} = \frac{\frac{q_0}{k} \pm \sqrt{(\frac{q_0}{k})^2 - 2\frac{A}{k}(T_c - T_0)}}{\frac{A}{k}}$$

This can be simplified $T(z) = \frac{q_0}{k}z + T_0$ set $T(z_b) = T_c$

$$(T_c - T_0) = \frac{q_0}{k}z_b$$

$z_b = \frac{k(T_c - T_0)}{q_0}$ this is the classic curie depth heat flow equation used in this magnetic spectral study. Since $G(z) = \frac{q_0}{k}$ -----(11)

we can connect curie depth heat flow and geothermal gradient.

$$z_b = \frac{(T_c - T_0)}{G}$$

$$q_0 = \frac{k(T_c - T_0)}{z_b} \text{-----(12)}$$

This implies that shallow curie depth signifies high heat flow and high geothermal gradient and deep curie depth signifies low heat flows and stable cratonic regions.

Results and Interpretation

The aeromagnetic data was converted to the frequency domain, and the radially averaged power spectra (Figure 4b) were used to estimate the depth to the top of magnetic sources (Z_t) and the centroid depth (Z_0). The results of these spectral depth parameters were then used to compute the Curie point depth (Z_b) and geothermal gradient (GT) and surface heat flow (SHF) and are summarized in Table 1. The values of depth to bottom, geothermal gradient and heat-flow were obtained from Equations 10, 11, and 12, respectively. The calculated parameters were then spatially interpolated to generate maps of depth to top, Curie point depth, geothermal gradient, and heat flow (Figures 5–7).

The upper boundaries of magnetic sources are seen to have a distinct spatial variation in the depth-to-top map (Figure 5a) where the blue–purple tones represent relatively shallow magnetic bodies while the orange hues represent relatively deeper magnetic sources. The estimated depth to the top of these thermomagnetic bodies gives an indication of the accessibility of possible geothermal resources. Across the study area, Z_t values range from 5.97 to 9.68 km, with an average of 7.34 km. The shallower source depths (~6.02 km) imply less need for drilling and improved economic potential for geothermal exploitation, while the deeper depths (~9.68 km) would necessitate more intensive drilling and costlier development.

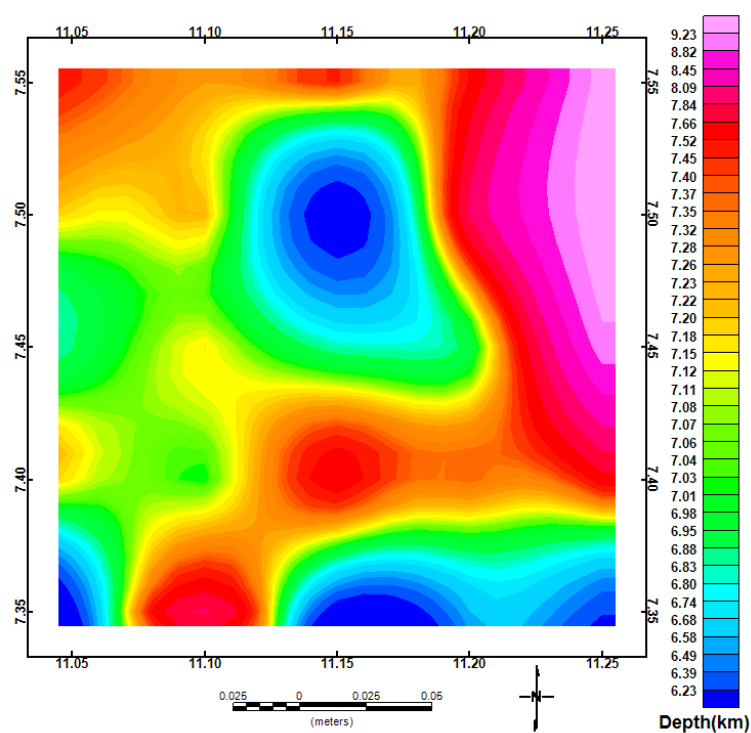


Figure 5a: Depth to Shallow Sources (computation from NGSA aeromagnetic data)

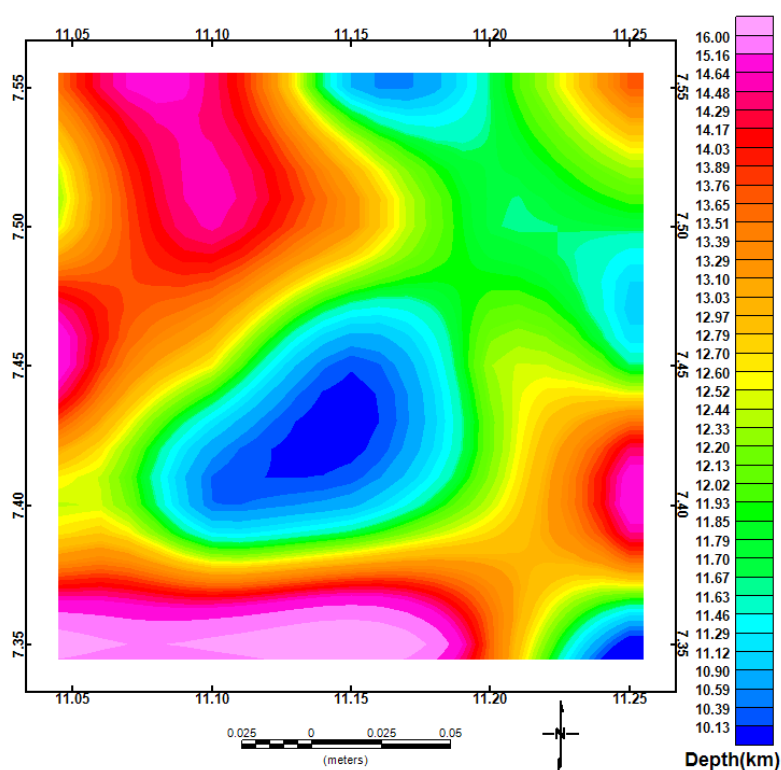


Figure 5b: Map of Depth to deep Sources (computation from NGSA aeromagnetic data)

The Depth to Centroid Map Figure 5b of the study area also showed varying depth with an average of 13km, with 9.7km as the least and 17.6km as the highest values. The highest depth to centroid is found at northwestern to southwest and the southern part of the study area depth above 14 km from on the map. While the low to lowest depth to deep sources are found trending northeast to southwestern part on the map.

Table 1: Estimated Curie point, Geothermal gradient and Heat flow from spectral analysis (computation from NGS Aeromagnetic data).

Blocks	Depth to deep sources z_o (km)	Depth to shallow sources z_t (km)	Curie Depth z_c (km)	Geothermal Gradient $dT/dZ(^{\circ}C/km)$	Heat flow $q(mWm^{-2})$
1	16.55	6.10	27.01	21.48	53.69
2	16.31	7.88	24.74	23.44	58.60
3	17.55	6.02	29.09	19.94	49.85
4	13.61	6.58	20.64	28.10	70.25
5	9.70	6.25	13.16	44.08	110.19
6	12.43	7.15	17.71	32.76	81.89
7	10.40	7.01	13.79	42.07	105.17
8	10.80	7.62	13.98	41.48	103.70
9	12.43	7.37	17.50	33.14	82.86
10	14.96	7.60	22.32	25.99	64.97
11	14.56	6.88	22.23	26.09	65.24
12	12.58	7.15	18.01	32.21	80.52
13	10.18	6.83	13.53	42.85	107.13
14	12.32	6.92	17.71	32.74	81.86
15	11.68	9.03	14.32	40.50	101.26
16	12.77	7.17	18.38	31.55	78.89
17	14.54	7.22	21.87	26.52	66.30
18	13.23	5.97	20.48	28.32	70.80
19	11.68	7.88	15.49	37.45	93.62
20	11.72	9.68	13.75	42.19	105.47
21	13.82	7.47	20.17	28.76	71.89
22	14.39	7.26	21.52	26.95	67.37
23	10.88	7.46	14.31	40.54	101.35
24	11.68	7.53	15.84	36.61	91.53
25	13.69	9.45	17.93	32.35	80.87

Curie Point Depth (CPD)

Figure 5c indicates the delineated CPD of the research region, the average depth in the study area is 18.6km with 13.2km and 29.1km as the lowest and highest values respectively. The delineated curie point depth of 20km and above dominates the Northwestern and southern flank where we have the Mayoselbe and Butale town. This signifies a cooler crustal regime with a low heat flow and lower geothermal energy potential. The low to lowest CPD below 20km is trending northeast to southwestern part cutting across Walu, Mayo-Juji, Galinzina, Ukwa, Mayo-Njim and Moubborn Hill above latitude $7.35^{\circ}N$ to $7.55^{\circ}N$ and above longitude $11.05^{\circ}E$ to $11.25^{\circ}E$. This signifies the curie isotherm is closer to the earth's surface.

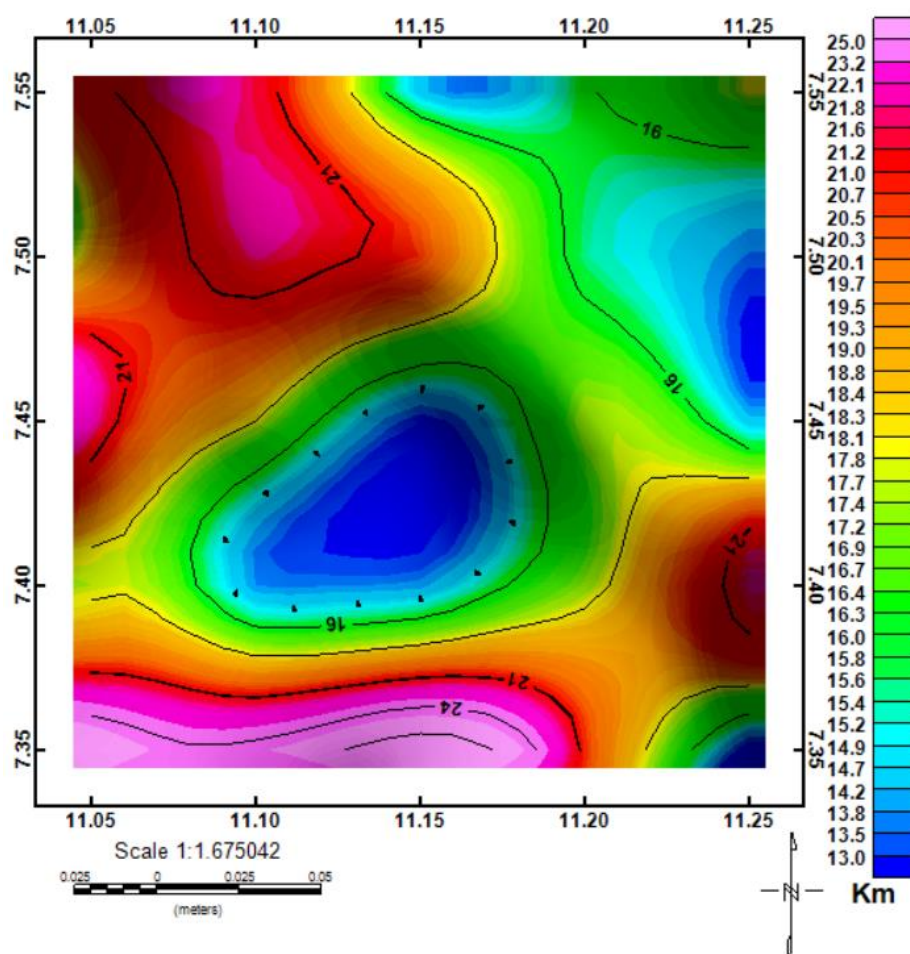


Figure 5c: Map of Curie Point Depth (computation from NGSa aeromagnetic data)

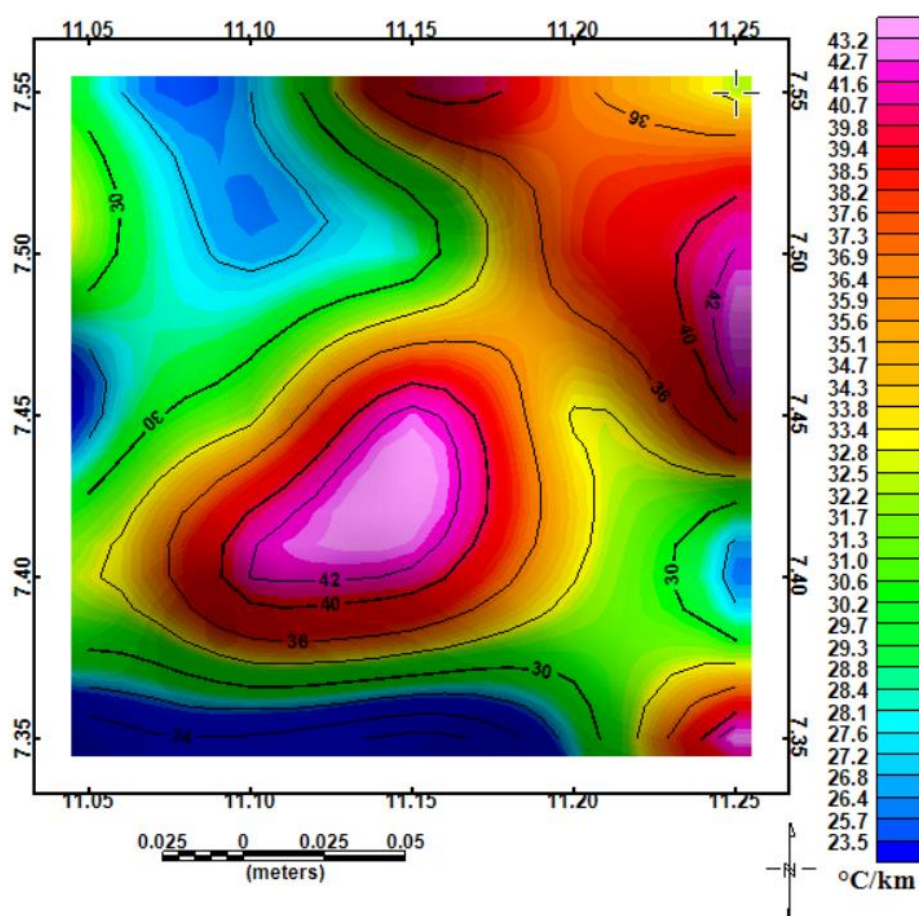


Figure 6: Geothermal Gradient Map (computation from NGSA aeromagnetic data)

Geothermal Gradient

The Geothermal Gradient Map (Figure 6) indicate that the geothermal gradient in ($^{\circ}\text{Ckm}^{-1}$) show the rate of depth dependent temperature growth. Figure 3 (e) showed this depth dependent temperature growth range 19.9°Ck/m to 44.1°Ck/m with an average value of 32.7°Ck/m . The high geothermal gradient indicates a thermally active crust with temperature increases rapidly with depth trending northeast-southwest cutting across Walu, Mayo-Juji, Galinzina, Ukwa, Mayo-Njim and Moubborn Hill above latitude 7.35°N to 7.55°N and above longitude 11.05°E to 11.25°E . The lowest values are found to be in the south and southwestern part of the study area in Mayoselbe and Butale town with a geothermal gradient range of 19.9°Ck/m to 26.4°Ck/m signifying that temperature increases slowly with depth often associated with deep curie point depths where magnetic minerals retain their magnetization at a greater depth.

Heat Flow (q)

The heat flow distribution map (Figure 7) exhibits a close spatial correspondence with the geothermal gradient map, where regions characterized by high, intermediate and low heat flow values coincides with zones of high, intermediate and low geothermal gradients. A general classification of the estimated heat-flow values is possible and is divided into three groups. The values of low heat-flow, 20–60 mW m⁻², are less than the expected range for stable continental crust. 60-100 mW m⁻² are in the normal continental geothermal regime. In contrast heat flow values exceeding 100 mW m⁻² surpass the average continental threshold and they are indicative of anomalous geothermal conditions found in regions with enhanced geothermal activity in the crust. The results also demonstrate that the northeast–southwest part of the study area has heat-flow values greater than this threshold, indicating the presence of high geothermal anomalies and favourable development of geothermal resources. The average heat flow value of 81.81 mW m⁻² for the study area suggests that the crustal thermal regime is transitioning towards a high geothermal state compared with continental environments.

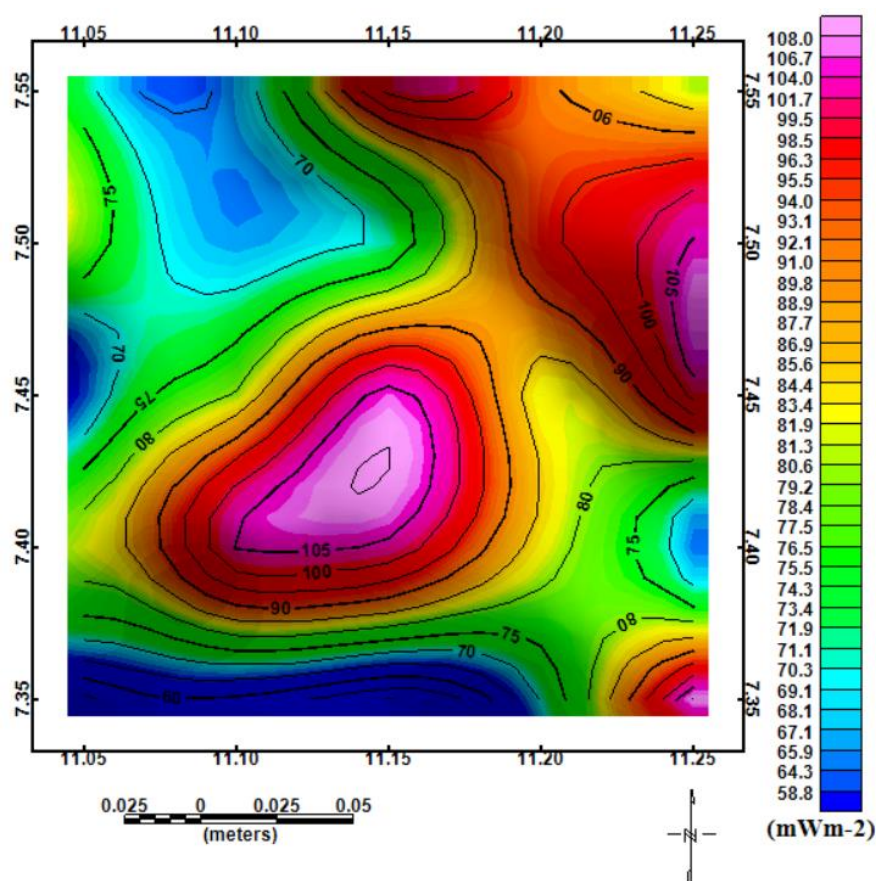


Figure 7: Heat flow Map (computation from NGSA aeromagnetic data)

Discussion**Why FFT is Necessary.****a) Spatial Data transformation into Frequency Domain**

Aeromagnetic anomalies are overlapping signals of two or more sources at different depths. The FFT transforms spatial data into the frequency (wavenumber) domain, allowing the data to be interpreted more easily, and signals can be separated by wavelength (Blakely, 1996; Maus et al., 2022). This transformation converts convolution operations into multiplication and hence, decreasing the complexity in computations involved during geophysical inversion (Li et al., 2022).

b) Reduction to Solvable Linear System

Aeromagnetic data is reduced to a solvable linear system whereby magnetic anomalies in the spectral domain decay exponentially with depth, allowing logarithmic transformation of the power spectrum into a linear relationship (Tanaka et al., 2021; Bouligand et al., 2022). It can be used to directly fit spectral curves to a straight line, estimate centroid and top depths and simplify the inversion by using regression methods. Therefore, FFT converts nonlinear geophysical equations to manageable linear equations (Maus and Dimri, 2021).

c) Hidden Periodicities Extraction.

FFT breaks down magnetic data into sinusoidal components, which identify concealed periodic patterns related to geological features, including faults, fractures, as well as intrusive bodies (Aboud et al., 2023; Nwankwo et al., 2022). This spectral decomposition allows distinguishing between shallow and deep sources in terms of content of frequencies (Salem et al., 2014).

Spectral Analysis: Why it Works.**a) Linear Superposition**

Magnetic fields obey the Linear Superposition, according to which the overall magnetic response is the sum of the responses of separate sources (Blakely, 1996). This enables spectral decomposition methods to remove the contributions of various depth levels (Maus et al., 2022).

b) Exponential Decay with Depth

Magnetic anomalies amplitude exponentially decreases with depth, so the theoretical basis of spectral depth estimation is established (Tanaka et al., 2021; Bouligand et al., 2022). This correlation suggests that high-frequency components have shallow sources, and low-frequency ones have deeper sources (Salem et al., 2014).

c) Statistical Stationarity

Spectral analysis makes an assumption of statistical stationarity of magnetic sources, that is, the statistical characteristics of magnetic sources do not change with space (Maus et al., 2021). This is an assumption that makes the power spectra stable and interpretable and enables accurate estimation of depth parameters even in complex geological settings (Li et al., 2022).

From Mathematics to Geoheat Parameters

Spectral analysis gives an opportunity to estimate the important geothermal parameters using magnetic data. The centroid depth and top boundary depth are calculated using various parts of the power spectrum and the Curie point depth is calculated based on the relationship between them (Tanaka et al., 1999; Nwankwo et al., 2022). Based on these parameters, the geothermal gradient and heat flow can be estimated, which gives an understanding of the thermal regime in the subsurface (Aboud et al., 2023).

Mathematics is made a geoheat engine as it turns variations in the magnetic field into thermal structure, Links frequency decay to subsurface temperature distribution. This transforms spectral energy hidden in the spectrum to geothermal information. Essentially, FFT and spectral analysis are not simply processing data that they convert the magnetic memory of the Earth into its thermal reality. Mathematics is also a geoheat engine as it converts the observations of aeromagnetic data into a quantitative estimate of the thermal structure of the subsurface, such as Curie depth, geothermal gradient, and heat flow (Salem et al., 2021; Li et al., 2022). The key elements of this transformation are the Fast Fourier Transform and Spectral Analysis that transform complex spatial geophysical signals into interpretable spectral representations (Maus and Dimri, 2021; Bouligand et al., 2022).

Conclusion

The spectral analysis based on FFT is a mathematically sound geophysical instrument since it lies between electromagnetism, Thermodynamics, Signal processing and allows indirect determination of Curie depth and Geothermal gradient and Heat flow. This mathematical representation indicates that FFT converts aeromagnetic data to physically interpretable spectral energy CPD can be estimated with spectral slope analysis with high reliability, Geothermal gradient and heat flow are both direct mathematical derivatives of CPD and the method gives a powerful, non-invasive geothermal exploration instrument. The intermediary between magnetic signal and the thermal structure of the earth is mathematics (FFT + spectral theory). In the absence of mathematics aeromagnetic data are noise, in the presence of FFT they are geothermal diagnostic tool.

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